Debt Maturity Structure with Pre-emptive Creditors

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Abstract

Critics of international solvency regimes argue that such regimes heighten financial fragility because creditors pre-empt each other by lending at ever shorter maturities in a 'rush for the exits'. We model such behaviour explicitly in order to examine the effects of workouts on the maturity profile of debt.

JEL Classification: F33; F34

Key words: Sovereign debt workouts; Debt maturity structure; International financial architecture.

1 Introduction

Statutory approaches to sovereign debt restructuring (e.g Krueger, 2002) have gained prominence in recent debates on the international financial architecture. Critics, however, argue that international solvency regimes exacerbate financial fragility by promoting shorter debt maturities. On this view, creditors have an incentive to 'rush for the exits', i.e. pre-emptively seek shorter and shorter maturities in the hope of avoiding being caught up in a payment suspension and the ensuing debt workout (see Geithner, 2000).

Debt maturity structure cannot be considered in isolation from the issue of pricing risky debt. In general, however, it is not possible to study the two simultaneously as the failure rate of a project and the pricing relationship are

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both endogenous and dependent on each other. Thus, reduced form credit models, such as Jarrow & Turnbull (1995), treat default as an exogenous event and take maturity profiles as given in order to price risky debt. This paper considers the complementary issue, namely the failure rate of a project implied by a given pricing structure, in order to examine the effects of workouts on the maturity profile of debt.

2 The model

A continuum of investors, each endowed with unit wealth, can choose to be equityholders or lend to the entrepreneur of a risky project. Those who lend must decide on the maturity of the debt contract, which can range from one period to T periods. The per capita value of the project at the initial date is 1. The value in the next period depends on the amount of debt that matures at that date. We suppose that the notional forward rate is constant and given by R, so that the notional yield on debt maturing at date t is given by R^t.[1]

The outcome of the project depends on the amount of short-term debt. The larger the amount of short-term debt due, the greater the probability that the project will fail in that period.[2] Thus, conditional on having succeeded up to date $t-1$, the project fails at date t with probability:

$$\gamma(t) \equiv \frac{p(t)}{p(T+1) + \sum_{s=t}^{T} p(s)} \tag{1}$$

where $p(t)$ is the size of the debt that matures at date t and $p(T+1)$ is the size of the equity holding.

[1] The assumption that the notional forward rate is constant is a convenient way to tie down the pricing relationship in order to focus on the determination of the failure rate (hazard rate), i.e. the probability that the borrower will default in period t, conditional on having survived till $t-1$.

[2] Rogoff (2003) stresses how the lack of an explicit seniority structure for sovereign debt means that investors impose risky (short-term) debt structures on the borrower to substitute for effective property rights at the international level. The empirical literature on the pricing of defaultable corporate debt also assumes a prominent role for short-term debt (see Crouhy et.al, 2001).

The longer the project continues, the greater is the break-up value of the project. If the project were to fail before the maturity of the project, liquidation costs are incurred which reduce the return to claimholders. So when the project fails between t and $t-1$, the project is liquidated for θR^t and all creditors receive this liquidation value. The parameter θ reflects the recovery rate facing the holders of debt. The equityholders receive nothing. But if the project survives date t, then lenders whose debt matures at date t receive the full notional value R^t. In order that short-term debt is not dominated by long-term debt, we impose the condition that $0 < \theta < 1/R$. If the project never fails and so succeeds at the terminal date T, then the value of the firm is W. Equityholders thus receive the residual payoff $W - R^T$ and all debtholders are paid in full. The payoffs of all the claimholders as a function of the date of the project failure can be represented in matrix form as

		Project failure date							
		1	2	3	4	5	$\cdots$	T	Never
	1	θR	R	R	R	R	$\cdots$	R	R
		θR	θR^2	R^2	R^2	R^2	$\cdots$	R^2	R^2
Debt	3	θR	θR^2	θR^3	R^3	R^3	$\cdots$	R^3	R^3
maturing	4	θR	θR^2	θR^3	θR^4	R^4	$\cdots$	R^4	R^4
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
	T	θR	θR^2	θR^3	θR^4	θR^5	$\cdots$	θR^T	R^T
Equity		0	0	0	0	0	$\cdots$	0	$W - R^T$

The action set of the individual is denoted by $\{1, 2, \cdots, T, T+1\}$, where $T+1$ indicates investing as an equityholder, while $t \le T$ indicates lending at maturity t. In general, the payoff to a particular action depends on how far the risky project progresses. Equity and longer maturity debt do better if the project reaches an advanced stage. For creditors, the payoffs have the feature that each creditor has an incentive to be "one step closer to the door" than other creditors, in the sense that if all other creditors are of maturity t, then the best reply is to choose maturity $t-1$. The only exception is when everyone chooses debt of maturity 1. In this case, creditors are indifferent between any maturity from 1 to T.

Normalizing the measure of investors to 1, let $p(t)$ be the measure of investors who take action t, so that the vector $[p(1), p(2), p(3), \cdots, p(T), p(T+1)]$ gives the capital structure of the risky project. To assess expected payoffs, we focus on the probability distribution over outcomes. The probability that the project fails at date t is given by

$$\begin{aligned} & [1-p(1)]\left[1-\frac{p(2)}{\sum_{s=2}^{T+1} p(s)}\right] \ldots\left[1-\frac{p(t-1)}{\sum_{s=t-1}^{T+1} p(s)}\right] \frac{p(t)}{\sum_{s=t}^{T+1} p(s)} \\ = & \sum_{s=2}^{T+1} p(s) \cdot \frac{\sum_{s=3}^{T+1} p(s)}{\sum_{s=2}^{T+1} p(s)} \cdots \frac{\sum_{s=t}^{T+1} p(s)}{\sum_{s=t-1}^{T+1} p(s)} \cdot \frac{p(t)}{\sum_{s=t}^{T+1} p(s)} \\ = & p(t) \end{aligned}$$

Thus, the expected payoff of each class of claimholder is obtained as the expectation of the payoff with respect to the probability density $[p(1),\ p(2),\ \cdots,\ p(T+1)]$. The expected payoff of the equityholder is

$$V(T+1) = p(T+1) \cdot (W - R^T), \tag{2}$$

while the expected payoff of the creditor with debt of maturity t is given by

$$V(t) = \sum_{s=1}^{t} p(s) R^{s-2} + R^t \sum_{s=t+1}^{T+1} p(s). \tag{3}$$

The equilibrium capital structure equalizes the expected payoff to each type of claimholder and is the capital structure for which

$$V(1) = V(2) = \cdots = V(T) = V(T+1). \tag{4}$$

Let M be the matrix of payoffs and p' be the row vector, $p' = [p(1),\ p(2)...,\ p(T),\ p(T+1)]$. Then the expected payoff to the claimholder of maturity t is given by the t-th entry of the vector:

$$Mp.$$

In order for all claimholders to have the same expected payoff, we must have

$$Mp = k$$

for some constant vector k. It can be verified that M is non-singular, so that the equilibrium capital structure p is obtained as

$$p = M^{-1}k, \tag{5}$$

where the elements of p sum to one.

Since the general solution to (6) is rather cumbersome, we focus on a parametric example. Specifically, when the recovery rate, θ, and project value, W, satisfy $\theta = 1/R^2$ and $W = (1+R)^T + R^T$, the equilibrium capital structure is described by:

Theorem 1 *The unique solution to (6) is*

$$p(t) = \begin{cases} R/(1+R)^t & if \quad t \leq T \\ 1/(1+R)^T & if \quad t = T+1 \end{cases}.$$

Moreover, the expected payoff to all investors is 1.

Proof. See appendix. ■

Since the expected payoff to all investors is one, any change in the notional yield curve is exactly offset by a commensurate change in the default rate. As R increases, there is a shift towards shorter maturity assets in the sense of first-degree stochastic dominance. Consider two notional forward rates R and $\widetilde{R}$, where $R < \widetilde{R}$, and let $P(t)$ be the cumulative distribution function corresponding to the density $p(.)$, i.e. $P(t) = \sum_{s \leq t} p(s)$. If $\widetilde{P}(t)$ is the corresponding cumulative distribution for $\widetilde{p}(.)$, then

Theorem 2 $P(t) \leq \widetilde{P}(t)$ *for all* t*, and strictly so for* $t \leq T$.

Proof. See appendix. ■

3 Conclusion

Our results suggest that international solvency regimes which improve the recovery rate on default may not necessarily imply a 'rush for the exits'. Developing a richer model that examines the effects of pre-emptive behaviour on both prices and maturities is an important area for future research.

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Appendix

Proof of Theorem 1

Let us first verify that $V(t) = 1$ for all $t \in \{1, 2, \cdots T+1\}$. For $t = T+1$, we have

$$V(T+1) = \left(W - R^T\right) \cdot \frac{1}{(1+R)^T} = 1$$

For $t \leq T$,

$$\begin{aligned} V(t) &= \sum_{s=1}^{t} p(s) R^{s-2} + R^t \sum_{s=t+1}^{T+1} p(s) \\ &= \sum_{s=1}^{t} \frac{R^{s-1}}{(1+R)^s} + R^{t+1} \sum_{s=t+1}^{T} \frac{1}{(1+R)^s} + \frac{R^t}{(1+R)^T} \end{aligned}$$

The two terms in the above equation can be re-expressed as $1 - \left(\frac{R}{1+R}\right)^t$ and $\left(\frac{R}{1+R}\right)^t \left(1 - \frac{1}{(1+R)^{T-t}}\right)$, so we have

$$\begin{aligned} V(t) &= 1 - \left(\frac{R}{1+R}\right)^t + \left(\frac{R}{1+R}\right)^t \left(1 - \frac{1}{(1+R)^{T-t}}\right) + \frac{R^t}{(1+R)^T} \\ &= 1 \end{aligned}$$

It remains to show that $\sum_t p(t) = 1$. From the definition,

$$\begin{aligned} \sum_{t=1}^{T+1} p(t) &= \tfrac{R}{1+R} \left(1 + \tfrac{1}{1+R} + \cdots + \tfrac{1}{(1+R)^{T-1}}\right) + \tfrac{1}{(1+R)^T} \\ &= 1 - \tfrac{1}{(1+R)^T} + \tfrac{1}{(1+R)^T} = 1 \qquad \blacksquare \end{aligned}$$

Proof of Theorem 2

Note that $R < \widetilde{R}$ implies that for any $s < t$,

$$\frac{p(s)}{p(t)} < \frac{\widetilde{p}(s)}{\widetilde{p}(t)}.$$

Summing over $s \in \{1, 2, ..., t-1\}$, we have

$$\frac{P(t-1)}{p(t)} < \frac{\widetilde{P}(t-1)}{\widetilde{p}(t)}.$$

Taking reciprocals and adding 1 to both sides,

$$\frac{P(t)}{P(t-1)} > \frac{\widetilde{P}(t)}{\widetilde{P}(t-1)}. \tag{6}$$

Since $P(T+1) = \widetilde{P}(T+1) = 1$, it implies that $P(T) < \widetilde{P}(T)$. We now have an inductive argument. Suppose that $P(t) < \widetilde{P}(t)$. We will show that $P(t-1) < \widetilde{P}(t-1)$. From (7) and the hypothesis that $P(t) < \widetilde{P}(t)$, we have

$$\frac{P(t)}{P(t-1)} > \frac{\widetilde{P}(t)}{\widetilde{P}(t-1)} > \frac{P(t)}{\widetilde{P}(t-1)}$$

so that $P(t-1) < \widetilde{P}(t-1)$. ■