Endogenous Disclosures and the Post Earnings Announcement Drift

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Motivation

- Short-run underreaction and long-run overreaction
- Critical role for disclosures
  - Not all “drift”
  - Adjustments delayed until subsequent earnings dates

Task: Integrate disclosures into asset pricing model

Ingredients

- Self-interested disclosures
- Sophisticated market
- Disclosures backed by necessary accompanying evidence (verifiable reports)

Binomial Model of Disclosures

N independent projects, success probability r

Liquidation value $u^s d^{N-s}$ for s successes, $N - s$ failures

- Three dates:
  - initial $(t = 0)$
  - interim $(t = 1)$
  - final $(t = 2)$

Probability θ that project outcome determined by interim date

Asymmetric information

Manager's **disclosure strategy**

$$(s, f) \mapsto (s', f') \qquad \text{where } (s', f') \leq (s, f) \quad \text{(verifiability)}$$

Market's **pricing strategy**

$$(s', f') \mapsto V_1$$

Manager maximizes V_1 (other prices V_0, V_2 based on symmetric information).

Market pegs price back to expected value given disclosure, i.e. minimize:

$$(V_1 - V_2)^2$$

Equilibrium (Bayes Nash, and refinements)

Full disclosure is never part of any equilibrium

Suppose it is. Then,

$$V_1(s,f) = u^s d^f (ru + (1-r)d)^{N-s-f}$$

But feasible disclosure $(s,0)$ yields

$$\begin{aligned} & u^s \sum_{i=0}^{N-s-f} \binom{N-s-f}{i} (ru)^i ((1-r)d)^{N-s-i} \\ = \quad & u^s (ru + (1-r)d)^{N-s} \end{aligned}$$

Contradiction.

Sanitization Strategy

$$(s, f) \longmapsto (s, 0)$$

Sanitization is optimal when pricing function is *monotonic* (i.e)

$$(s, -f) \geq (s', -f') \quad \Rightarrow \quad V_1(s, -f) \geq V_1(s', -f')$$

But note: (discussed in Shin (2003))

- there are monotonic equilibria without sanitization
- there are non-monotonic equilibria

Write $V_1(s) = V_1(s, 0)$

Posterior Density

actual successes

disclosed successes		0	$\cdots$	j	$\cdots$	N
	0					
	$\vdots$					
	s			$h(s, j)$		
	$\vdots$					
	N					

$$h\left(j|s\right) = \begin{cases} \binom{N-s}{j-s} q^{j-s} \left(1-q\right)^{N-j} & \text{if} \quad s \leq j \\ 0 & \text{otherwise} \end{cases}$$

where $q = \left(r - \theta r\right) / \left(1 - \theta r\right).$

Concatenation of Binomial Trees

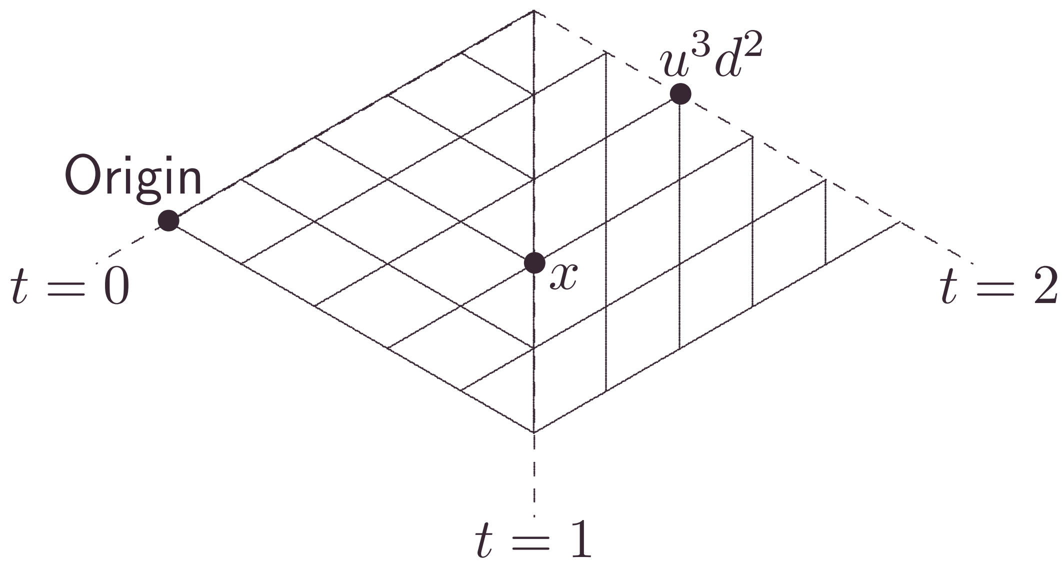


Figure 1: Concatenation of Binomial Trees

Poisson Model of Disclosures

Variant of binomial model

- Disclosures at dates $1, 2, \cdots, t, \cdots, T$
- $\theta_t = t\theta$
- $d = 1$
- $N \to \infty, r \to 0$ but $rN \to \lambda$

Residual trees are always of the same size (i.e. infinite)

Recursive structure ...

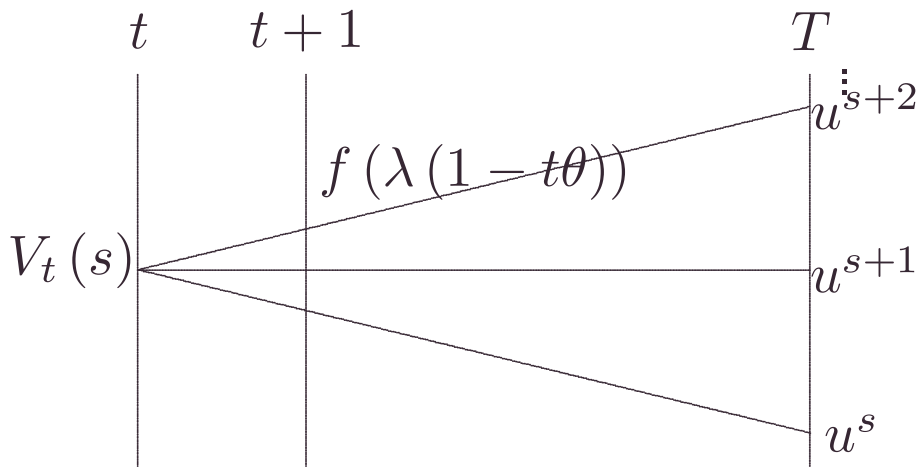
Recursive Structure

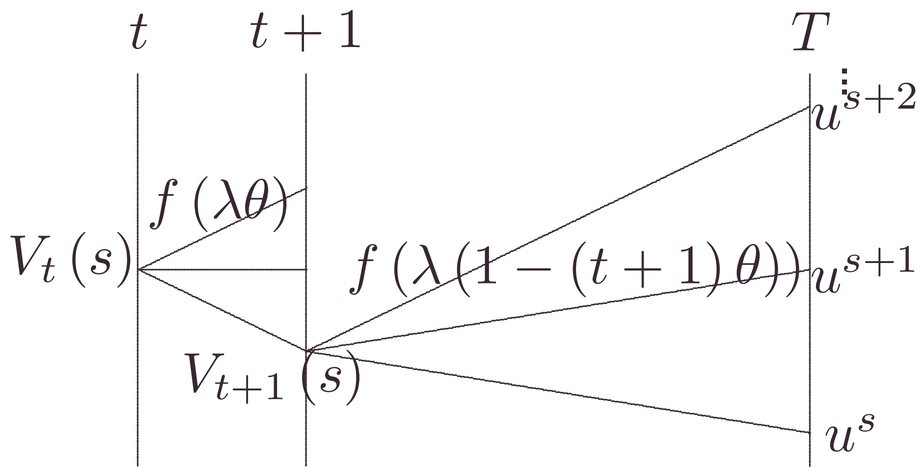
a-Poisson matrix

$$P(a) \equiv e^{-a} \begin{bmatrix} 1 & a & \frac{a^2}{2!} & \frac{a^3}{3!} & \cdots \\ 0 & 1 & a & \frac{a^2}{2!} & \cdots \\ 0 & 0 & 1 & a & \cdots \\ 0 & 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Then,

$$P(a)\,P(b) = P(a+b)$$


t
$t+1$
T
u^{s+2}
$f(\lambda(1-t\theta))$
$V_t(s)$
u^{s+1}
u^s


t
$t+1$
T
u^{s+2}
$f(\lambda\theta)$
$V_t(s)$
$f(\lambda(1-(t+1)\theta))$
u^{s+1}
$V_{t+1}(s)$
u^s

Prices and Returns with Risk-neutrality

$$V_t(s) = u^s \exp\{\lambda(1-t\theta)(u-1)\}$$

$$\begin{aligned} Var_t(R_{t,t+1}) &= \exp\left\{\lambda\theta(u-1)^2\right\} - 1 \\ Var_t(R_{t,T}) &= \exp\left\{\lambda(1-t\theta)(u-1)^2\right\} - 1 \end{aligned}$$

where $R_{t,t+1} = \dfrac{V_{t+1}}{V_t}$.

Different effect of θ on short and long-run return volatility...

Returns with Risk Aversion

Constant relative risk-aversion α.

Equivalent martingale measure: replace λ by λ/u^{α}.

$$\begin{aligned} E_t\left(R_{t,t+1}|s\right) &= \exp\left\{\lambda\theta\left(u-1\right)\left(1-\frac{1}{u^{\alpha}}\right)\right\} \\ E_t\left(R_{tT}|s\right) &= \exp\left\{\lambda\left(1-t\theta\right)\left(u-1\right)\left(1-\frac{1}{u^{\alpha}}\right)\right\} \end{aligned}$$

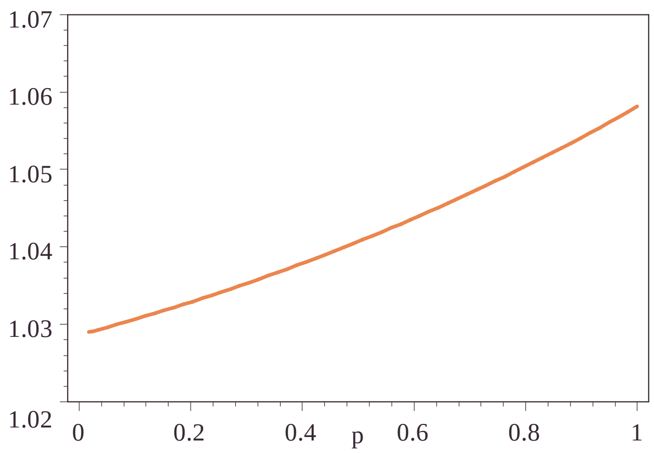
Theorem. Short-run return is increasing in θ, but long-run return is decreasing in θ.

Uncertainty over θ

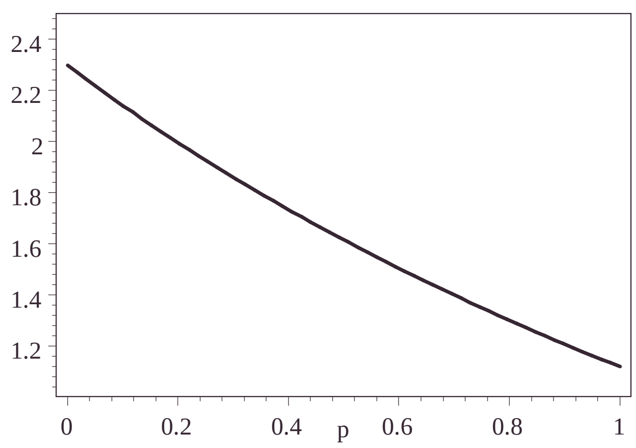
$\theta \in \{\theta_L, \theta_H\}$

p_j, posterior weight on θ_H given disclosure j

p_j is increasing in j.



Short-run expected return



Long-run expected return

Closed Form Solution

$\xi \equiv \lambda\theta$ has gamma prior

$$f(\xi) = \frac{1}{K}\xi^{\sigma}e^{-\tau\xi}$$

Updated density after s disclosed successes at date t

$$g(\xi|s,t) = \frac{1}{\hat{K}}\xi^{\sigma+s+1}e^{-\xi(\tau+t)}$$

Gamma and Poisson are conjugates (De Groot (1970)).

Short-run expected return $E_t\left(R_{t,t+1}|s\right)$

$$\simeq \left(\frac{\tau + t\left(1+\frac{u-1}{u^\alpha}\right)}{\tau + t\left(1+\frac{u-1}{u^\alpha}\right) - (u-1)\left(1-\frac{1}{u^\alpha}\right)}\right)^{1+\sigma+s}$$

Long-run expected return $E_t\left(R_{t,T}|s\right)$

$$\simeq e^{\lambda(u-1)\left(1-\frac{1}{u^\alpha}\right)} \cdot \left(\frac{\tau + t\left(u - (u-1)\left(1-\frac{1}{u^\alpha}\right)\right)}{\tau + tu}\right)^{1+\sigma+s}$$

These approximations are exact in the limit as $\lambda \to \infty$.

Summing up

- Asset prices determined by *residual* uncertainty.

  Short-run returns determined (in part) by uncertainty over *future* disclosures.

  Empirical hypothesis: firms with positive earnings surprises have more volatile subsequent earnings

Disclosures display complex patterns over time (e.g. Foster (77))

Information is not exogenous