

DISCLOSURES AND ASSET RETURNS ¹

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Abstract: Public information in financial markets often arrives through the disclosures of interested parties who have a material interest in the reactions of the market to the new information. When the strategic interaction between the sender and the receiver is formalized as a disclosure game with verifiable reports, equilibrium prices can be given a simple characterization in terms of the concatenation of binomial pricing trees. There are a number of empirical implications. The theory predicts that the return variance following a poor disclosed outcome is higher than it would have been if the disclosed outcome were good. Also, when investors are risk averse, this leads to negative serial correlation of asset returns. Other points of contact with the empirical literature are discussed.

KEYWORDS: Disclosure games, residual uncertainty, binomial trees.

1 Introduction

The arrival of public information influences asset prices, but frequently such information is provided by interested parties who have a material interest in the reactions of the market to such news. Managers of a firm will consider carefully the consequences of any disclosure for the stock price of the firm. Governments, monetary authorities and other public bodies often have an interest in presenting a case to the market that fosters their intended goals. This is especially true in times of financial crises, when market reactions have a large and disproportionate impact on the direction of policy. What are the consequences for the time series of asset returns when public information is provided by interested parties? How are the interests of the various claim holders affected when information is provided in this way?

We address these questions in a disclosure game with verifiable reports, introduced by Grossman (1981) and Milgrom (1981). The game studies the interaction between an interested party (such as the manager of a firm) and sophisticated market traders. The manager aims to maximize the market price of the firm but the market's inference pegs the price of the firm to its actuarially fair value given the manager's disclosure strategy. The manager has more information about the true value of the firm, and is mandated to make reports to the market (such as through periodic earnings reports) in which any claim is supported by the necessary accompanying evidence. Disclosures are verifiable in this sense. The market's response is to discount any disclosure appropriately given the degree of asymmetry of information, and

the actual disclosure received. The uncertainty facing the market is therefore the residual uncertainty remaining after the self-interested disclosure by the interested party has been discounted by the market. We will highlight the characteristic features of such residual uncertainty and contrast them with the residual uncertainty following the exogenous arrival of information.

By drawing attention to the pricing implications of self-interested disclosures, the objective of this paper is to provide firmer foundations for empirical research and for policy debates on the incentives and institutions that underpin information dissemination in financial markets. The long bull run in the stock market in the 1990s has thrown into sharper relief the potential for abuse of the flexibility and discretion in accounting disclosures that were originally intended to allow financial reports to keep pace with innovations in business practices and transactions. In a hard-hitting statement, Arthur Levitt the then chairman of the U.S. Securities and Exchange Commission decries the “numbers game” played by the major market participants in this abuse (Levitt (1998)). These concerns proved to be well-founded subsequently with the revelations from the Enron affair.

Indeed, the stock price reactions to disclosures can sometimes seem bizarre unless the incentives for disclosure are taken into account. During the corporate reporting season, the well-publicized “official” consensus forecasts obtained from analysts are seen as susceptible to influence by the management in an effort to downplay expectations. Of more importance to the canny market participants are the unofficial “whisper numbers” which capture the true expected forecasts of the market. The late 90s bull market spawned web

sites dedicated to publicising these whisper numbers², and they document evidence of sharp falls in stock prices when corporate earnings disclosures fell short of the whisper numbers, even though they comfortably exceeded the official forecasts. One often witnesses the spectacle of commentators rationalizing such movements in terms of the announcement “not exceeding expectations by enough”. An analysis of the incentives of analysts would take us too far afield and is not attempted here. However, by studying the incentives underpinning managers’ disclosures, we provide the necessary foundations for any development in this direction.

In choosing to analyse disclosures in terms of the verifiable reports framework rather than the alternative “cheap talk” framework popularized by Crawford and Sobel (1982), we attempt to capture the twin themes of the discretion that managers have in their disclosures, but also the broad limits imposed by the accounting system on what is possible, with the ultimate sanction being the one against fraud. In this respect, we follow earlier studies in the accounting literature (see, for example, Verrecchia (1983) and Dye (1985a, 1985b)). Okuno-Fujiwara, Postlewaite and Suzumura (1990) discuss the broader institutional issues underlying the assumption of verifiability. They note, for instance, the effectiveness of being able actually to exhibit the evidence in support of a claim that one knows a particular feature of the world, and contrast this with the difficulty of proving the negative - that one is ignorant of some feature of the world. Bull and Watson (2000) bolster this intuition by showing that coalition-proof allocations rest on the ability of players to exhibit “positive evidence” of this kind, rather than simply on

the players' information partitions.

Our framework yields an attractively simple characterization of the way that uncertainty is resolved over time when public information arrives through self-interested disclosures. Building on the device of binomial pricing trees introduced by Cox, Ross and Rubinstein (1979), we show how the resolution of uncertainty can be represented by paths in a *concatenation* of binomial pricing trees. This representation gives us a simple pricing rule that can be used to address empirical issues concerning the time series properties of asset returns. In particular, the theory generates the following prediction. Following a 'bad' disclosed outcome, the uncertainty over subsequent outcomes increases. A number of related topics in the empirical literature rest on this feature of the time series of returns. Fisher Black first documented how low stock return was associated with an increase in the subsequent return volatility, and suggested the hypothesis that the reduced proportion of equity within the total assets of the firm may be one explanation for this empirical regularity (Black (1976)). The terminology of Black's "leverage hypothesis" has increasingly come to denote the empirical regularity itself, rather than the hypothesis originally put forward by Black, and has been the subject of a voluminous econometric literature. However, much remains to be learned of the underlying microeconomics of this phenomenon. Our framework may throw some light on this issue.

A related topic is the pricing of defaultable securities. Since the payoff to a creditor is akin to having a short position in a put option on the assets of the firm, with a strike price equal to the face value of the debt, option

pricing techniques can be used to determine the price of debt. This was the contribution of Merton's (1974) classic paper. Nevertheless, the empirical success of this approach has been mixed, with the usual discrepancy appearing in the form of the overpricing (by the theory) of the debt, and especially of the lower quality, riskier debt. To the extent that the payoff function to a creditor is akin to a short position in a put option, while the payoff of the equity holder is akin to a long position in a call option, any increase in underlying volatility implies a transfer away from the creditor in favour of the equity holder. As compared to the benchmark case in which volatility is constant, the increased volatility following a bad outcome for the firm lowers the price of debt by *more* than the benchmark case, and lowers the price of equity by *less* than the benchmark. This and other points of contact with the empirical literature are developed in greater detail in what follows.

The outline of our paper is as follows. We begin in the next section by presenting the basic formal model of disclosures. After characterizing the resolution of uncertainty in terms of the concatenation of binomial trees in section 3, we turn to the points of contact with the empirical literature in section 4. In section 5, we review the robustness of our main conclusions by examining a possible extension of our model where a firm has an unbounded number of projects. Section 6 concludes.

2 Model

Our framework builds on the binomial tree model of Cox, Ross and Rubinstein (1979), which has found widespread application in the option pricing

literature. A firm undertakes N independent and identical projects, where each project succeeds with probability r , and fails with probability $1 - r$. The liquidation value of the firm given s successes and $N - s$ failures is given by

$$(1) \quad u^s d^{N-s}$$

where $0 < d < u$. Each success corresponds to an “up” move in a tree that raises total liquidation value by a factor of u , while each failure corresponds to a “down” move of factor d . The ex ante value of the firm, denoted by V_0 , is the expected liquidation value obtained from the binomial density with success probability r . Thus,

$$\begin{aligned} V_0 &= \sum_{s=0}^N \binom{N}{s} (ru)^s ((1-r)d)^{N-s} \\ &= (ru + (1-r)d)^N \underbrace{\sum_{s=0}^N \binom{N}{s} \left[\frac{ru}{ru+(1-r)d} \right]^s \left[\frac{(1-r)d}{ru+(1-r)d} \right]^{N-s}}_{=1} \\ &= (ru + (1-r)d)^N. \end{aligned}$$

There are three dates - *initial*, *interim* and *final*. We index these dates by 0, 1 and 2 respectively. At the initial date, nothing is known about the value of the firm other than the description above. As time progresses, the projects begin to yield their outcomes. At the interim date, not enough time has elapsed for the manager to know the outcomes of all the projects. However, the outcomes of *some* of the projects will have been realized. In particular, there is a probability θ that the outcome of a project is revealed to the manager by the interim date. This probability is identical across all projects, and whether the outcome is revealed is independent across projects.

By the final date all uncertainty is resolved. The outcomes of all the projects become common knowledge. The firm is liquidated, and consumption takes place.

There is differential information at the interim date between the manager of the firm and the rest of the market. The manager is able to observe the success and failure of each project as it occurs, and hence knows the numbers of successes and failures at the interim date, but the rest of the market does not. Instead, the only information available to the market at the interim date is a *disclosure* by the manager. The manager is free to disclose some or all of what he knows, by actually exhibiting the outcome of those projects whose outcomes have already been determined. However, he cannot concoct false evidence. If he knows that project j has failed, he cannot claim that it has succeeded. In this sense, although the manager has to tell the truth, he cannot be forced to tell the whole truth.

The implicit understanding is that the manager's disclosures are verifiable at a later date by a third party, such as a court, who is able to impose a very large penalty if the earlier disclosure is exposed to be untrue, i.e. inconsistent with evidence made available by the manager. But how much private information the manager has at the time of disclosure is not verifiable even at a later date. So the manager is free to withhold information if such information is deemed to be unfavourable.

More formally, the information available to the manager at the interim date can be summarized by the pair (s, f) , where s is the number of successes observed and f is the number of failures observed. The manager's disclosure

strategy $m(\cdot)$ maps his information (s, f) to the pair (s', f') , giving the number of *disclosed* successes and failures, where the requirement of verifiability imposes the constraint that

$$(2) \quad s' \leq s \quad \text{and} \quad f' \leq f.$$

This constraint reflects the requirement that the disclosure takes the form of actually exhibiting a subset of the realized outcomes to the market.

We assume that the disclosure policy of the manager is motivated by the objective of maximizing the price of the firm. Since the initial and final price of the firm is based on symmetric information, the focus of the analysis will be on the interim price V_1 . The market, however, anticipates the manager's disclosure policy, and prices the firm by discounting the manager's disclosures appropriately. This gives rise to a game of incomplete information. We will model the "market" as a player in the game who sets the price of the firm to its actuarially fair value based on all the available evidence, taking into consideration the reporting strategy of the manager.

More formally, the market's strategy is the pricing function

$$(3) \quad (s', f') \mapsto V_1$$

We ensure that the market aims to set the price of the firm to its actuarially fair value by assuming that its objective in the game is to minimize the squared loss function:

$$(4) \quad (V_1 - V_2)^2$$

where V_2 is the (commonly known) liquidation value of the firm at the final date. The market then sets V_1 equal to the expected value of V_2 conditional on the disclosure of the manager, as generated by his disclosure strategy. The manager, on the other hand, anticipates the optimal response of the market, and chooses the disclosure that maximizes V_1 .

3 Equilibrium

One immediate conclusion we can draw is that a policy of full disclosure by the manager can never be part of any equilibrium. To see this, suppose for the sake of argument that the manager always discloses fully, so that the disclosure strategy is the identity function:

$$m(s, f) = (s, f).$$

The best reply by the market is to set V_1 to be

$$V_1(s, f) = u^s d^f (ru + (1 - r)d)^{N-s-f},$$

since there are $N - s - f$ unresolved projects, and the expected value of the firm is

$$\begin{aligned} & u^s d^f \sum_{i=0}^{N-s-f} \binom{N-s-f}{i} (ru)^i ((1-r)d)^{N-s-f-i} \\ (5) \quad & = u^s d^f (ru + (1-r)d)^{N-s-f}. \end{aligned}$$

But then, the manager's disclosure policy is sub-optimal, since the feasible disclosure $(s, 0)$ that suppresses all failures elicits the price:

$$(6) \quad u^s (ru + (1 - r)d)^{N-s},$$

which is strictly higher than (5) for positive f . Hence, we are led to a contradiction if we suppose that full disclosure can figure in an equilibrium of the disclosure game.

Having ruled out full disclosure, a natural place to turn next is to go the opposite extreme and consider the strategy in which all successes are disclosed, but none of the failures are disclosed. This is the strategy that maps (s, f) to $(s, 0)$, and we could dub it the *sanitization strategy* in that the disclosure is “sanitized” by removing the bad news but leaving all the good news. We will show that this strategy can, indeed, be supported in equilibrium, and will derive the observable implications for equilibrium prices. Under the constraint of verifiability, the sanitization strategy would be optimal for the manager whenever the pricing rule is *monotonic* in the sense that, if $(s, -f) \geq (s', -f')$ then

$$V_1(s, f) \geq V_1(s', f').$$

Monotonicity is arguably a very natural condition in the context of a disclosure game, and provides strong motivation for studying the equilibrium prices that result from the sanitization strategy. However, it should be borne in mind that there are equilibria that violate monotonicity. Also, a monotonic pricing rule may be consistent with a strategy that does not sanitize. Appendix A presents two examples. In the first, the manager discloses some

failures in equilibrium even though the pricing rule is monotonic, while in the second example, the equilibrium pricing rule fails monotonicity³.

The construction of these examples serve as reminders of the difficulty of tying down beliefs in sequential games of incomplete information. Although these counterexamples to sanitization are instructive, they do not detract from the appeal of the simplicity and intuitive force of the sanitization strategy, especially for the purpose of drawing empirical hypotheses. For this reason, we will confine our attention in what follows to equilibria in which the manager uses the sanitization strategy. Since failures are never disclosed under the sanitization strategy, the interim prices that occur with positive probability are of the form $V_1(s, 0)$, which we denote simply by $V_1(s)$.

3.1 Sequential Resolution of Uncertainty

The partial revelation of information that arises from the manager's disclosure at the interim date suggests a parallel between our problem of pricing the firm and the pricing of a compound lottery. The similarity lies in the fact that, in both cases, the uncertainty is resolved in two steps. At the interim date, the manager's disclosure still leaves some residual uncertainty in the true value of the firm, so that the firm at the initial date (date 0) is like a compound lottery where the prizes at the interim date (date 1) are also lotteries over prizes at the final date (date 2). However, when the manager follows the sanitization strategy, the uncertainty from date 0 to date 1 takes a very simple form. We know that the probability density over the true number of successes is binomial with probability r . When the manager follows

the sanitization strategy, the density over *disclosed successes* at date 1 is also binomial, with success probability θr . This is because the manager observes the success of any particular project with probability θr , and observations of successes are independent across projects.

Of interest is the residual uncertainty conditional on the manager's disclosure at date 1. Consider the joint density over *disclosed* successes at date 1 and the *realized* successes at date 2 when the manager follows the sanitization strategy. We can depict this density in tabular form as below.

		realized successes at $t = 2$				
		0	...	k	...	N
disclosed	\vdots					
successes	s			$h(s, k)$		
at $t = 1$	\vdots					
	N					

Since the number of disclosed successes cannot exceed realized successes, all entries below the leading diagonal are zero. $h(s, k)$ is the probability that the manager discloses s successes at date 1 when the realized number of successes at date 2 turns out to be k . Denoting by

$$h(k|s)$$

the probability of k realized successes conditional on disclosure of s successes, we have the following result.

Lemma 1 *Let $q = (r - \theta r) / (1 - \theta r)$. When the manager follows the sani-*

tization strategy,

$$(7) \quad h(k|s) = \begin{cases} \binom{N-s}{k-s} q^{k-s} (1-q)^{N-k} & \text{if } s \leq k \\ 0 & \text{otherwise} \end{cases}$$

In other words, the residual uncertainty can also be characterized by a binomial density in which the probability of success of an undisclosed project is given by $q = (r - \theta r) / (1 - \theta r)$. Pursuing the analogy with compound lotteries further, the lottery at date 0 is governed by a binomial density over prizes at date 1, where the prizes are themselves lotteries governed by binomial densities. The proof of the lemma is given in appendix B.

The recursive nature of the uncertainty underlying our problem implied by lemma 1 makes possible a convenient diagrammatic device to represent the resolution of uncertainty over time. Refer to figure 1 for an example of a firm with five projects.

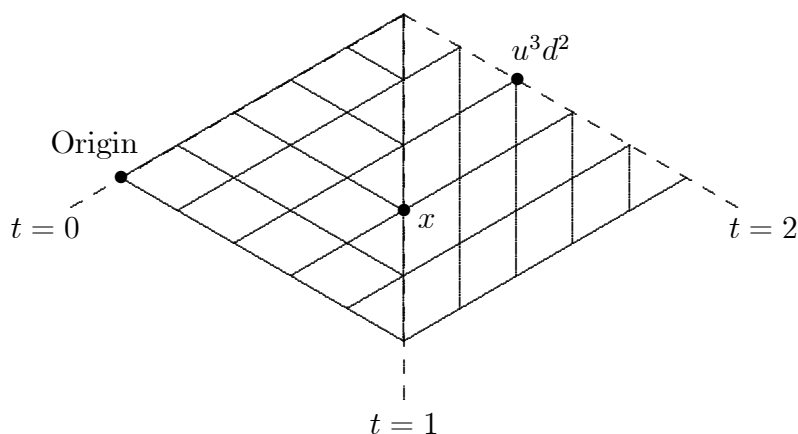


Figure 1: Concatenation of Binomial Trees

The starting point at date 0 is indicated as the origin in figure 1. Between

date 0 and date 1, uncertainty is resolved according to a binomial tree in which the probability of an up move is $r\theta$ while the probability of a down move is $1 - r\theta$. The point x represents the point reached at date 1 when the manager has disclosed two successes. Lemma 1 tells us that at point x , the residual uncertainty over the three undisclosed projects is governed by a binomial density with success probability q . This is represented by the binomial tree of length 3 with x at its apex, where an up move has probability q and a down move has probability $1 - q$. One possible outcome starting from x is for the eventual number of successes at date 2 to be exactly three. In terms of figure 1, this is represented by the fact that the terminal node u^3d^2 forms part of the binomial tree with root x . Finally, when viewed from the origin at date 0, the final density over the terminal nodes at date 2 is given by the binomial density with success probability r .

Lemma 1 reflects a closure property of binomial densities in which a weighted average of densities with binomial weights is also a binomial density. To state the closure property more formally and to understand better the comparative statics of information, let us introduce the notion of a *q-binomial matrix*. This is an upper triangular matrix of order $N + 1$ whose (s, k) -th entry (counting from zero) is given by (7). In other words, the *q*-binomial matrix is an upper triangular matrix with the property that the s th row of the matrix starts with a sequence of zeros until we reach the leading diagonal. Thereafter, the s th row consists of the probabilities of the binomial density with $N - s$ trials with success probability q . Let us denote the *q*-binomial

matrix as:

$$(8) \quad B(q)$$

Binomial matrices satisfy the following closure property.

Lemma 2 $B(p) B(q) = B(p + q - pq)$

Proof. The (i, s) th entry of $B(p) B(q)$ is given by

$$\begin{aligned} & \sum_{j=i}^s \binom{N-i}{j-i} p^{j-i} (1-p)^{N-j} \binom{N-j}{s-j} q^{s-j} (1-q)^{N-s} \\ &= [(1-p)(1-q)]^{N-s} \sum_{j=i}^s \underbrace{\binom{N-i}{j-i} \binom{N-j}{s-j}}_{=\binom{N-i}{s-i} \binom{s-i}{j-i}} p^{j-i} [(1-p)q]^{s-j} \\ &= \binom{N-i}{s-i} [(1-p)(1-q)]^{N-s} (p + (1-p)q)^{s-i} \underbrace{\sum_{j=i}^s \binom{s-i}{j-i} \left[\frac{p}{p+(1-p)q} \right]^{j-i} \left[\frac{(1-p)q}{p+(1-p)q} \right]^{s-j}}_{=1} \\ &= \binom{N-i}{s-i} (p + q - pq)^{s-i} [(1-p)(1-q)]^{N-s}, \end{aligned}$$

which is the (i, s) th entry of $B(p + q - pq)$. This proves the lemma.

Since $0 \leq p + q - pq \leq 1$ for any probabilities p and q , binomial matrices are closed under multiplication. In particular, for $p = r\theta$ and $q = (r - r\theta) / (1 - r\theta)$, we have

$$(9) \quad B(r) = B(p) B(q).$$

Since $p = r\theta$, the distribution over *disclosed successes* is given by the top row of $B(p)$. The top row of $B(r)$ gives the ex ante distribution over successes. Since the ex ante distribution over successes must be equal to the

average of the posterior distributions weighted by the probability of each disclosure, equation (9) confirms the result stated in lemma 1 that the s th row of $B(q)$ gives the distribution over realized successes conditional on s disclosed successes.

Note that $B(q)$ tends to the identity matrix as $\theta \rightarrow 1$. This has a natural interpretation. When θ is large, the manager is well informed about the true number of successes, and the disclosure is informative. In the limit, the manager is fully informed, so that there is full revelation of the ex post number of successes. The market's response is to put all the weight on the worst possible outcome consistent with the manager's disclosure. This is the so-called "unravelling" argument discussed in Milgrom (1981), Grossman (1981) and Milgrom and Roberts (1986), and much of the literature has focused on the conditions under which full revelation takes place (such as Lipman and Seppi (1995), Seidmann and Winter (1997)). However, as long as $\theta < 1$, the manager is not always fully informed, and the market must make allowance for some pooling between genuinely uninformed types of the manager and those types that are fully informed but are withholding information⁴.

At the opposite extreme to the case of full revelation, we have the case in which $\theta = 0$. Then, we have $B(q) = B(r)$, so that the conditional density is obtained from the prior probability of success given by r . The market thus takes the manager's disclosures at face value. For intermediate values of θ , the market discounts the manager's disclosures reducing the probability q in the pricing formula. When $\theta = 0$, only one disclosure receives positive weight,

namely the one in which no successes are reported. Any favourable shift in the disclosure of successes (an increase in p) is matched by a corresponding opposite shift in the posterior probability of success (a decrease in q). The market discounts the opportunistic disclosures of the manager by adjusting downwards the probability of eventual success at the final date. For any given r (ex ante probability of success), the trade-off between p and q is given by

$$(10) \quad \frac{dq}{dp} = -\frac{1-q}{1-p}.$$

3.2 Equilibrium Prices

By pricing the residual uncertainty from date 1 to date 2, we can characterize equilibrium prices at date 1.

Theorem 3 *There is a sequential equilibrium in which the manager uses the sanitization strategy. Moreover, in any equilibrium in which the manager uses the sanitization strategy,*

$$(11) \quad V_1(s) = u^s (qu + (1-q)d)^{N-s},$$

where $q = (r - \theta r) / (1 - \theta r)$.

Proof. The second statement follows directly from lemma 1 since

$$(12) \quad \begin{aligned} V_1(s) &= u^s \sum_{k=s}^N h(k|s) u^k d^{N-k} \\ &= u^s \sum_{i=0}^{N-s} \binom{N-s}{i} (qu)^i ((1-q)d)^{N-s-i} \\ &= u^s (qu + (1-q)d)^{N-s}. \end{aligned}$$

To complete the proof, we construct an equilibrium of the disclosure game where the manager follows the sanitization strategy. The solution concept is the notion of sequential equilibrium due to Kreps and Wilson (1982). The equilibrium pricing rule must specify a price for all feasible reports (s, f) , not simply those that receive positive probability in equilibrium. In addition, the beliefs given these out of equilibrium disclosures must be consistent with the rules of the game and be obtained as the limit of a sequence where each feasible disclosure receives positive probability from some type that “trembles” and discloses the out of equilibrium report by mistake.

The following pair of strategies are supported in a sequential equilibrium. The manager follows the sanitization strategy, while the market’s pricing rule is given by

$$V_1(s, f) = \begin{cases} u^s (qu + (1 - q)d)^{N-s} & \text{if } f = 0 \\ u^s d^{N-s} & \text{if } f > 0 \end{cases}$$

The off-equilibrium prices are supported by the belief that any disclosed failures emanate from the manager who knows the true value of the firm perfectly, and for whom the number of successes is exactly the number reported by him (so that all other projects are known to have failed). The sanitization strategy is clearly the best reply against this pricing strategy, and the pricing strategy is a best reply to the sanitization strategy. This concludes the argument.

3.3 Multi-period Extension

The closure property of binomial matrices and the recursive nature of the resolution of uncertainty in our framework enables us to extend the concatenation of binomial trees to a multi-period setting with several disclosure stages. Let us consider a model with $T + 1$ periods, indexed by the set $\{0, 1, 2, \dots, T\}$. Date 0 is the initial date, and date T is the terminal date when all uncertainty is resolved. At the interim dates $1, 2, \dots, T - 1$, the manager makes a verifiable disclosure on the *cumulative number* of successes to date. There are N projects in total, each with success probability r .

The equilibrium selection problem is undoubtedly be more acute in a multi-period setting. As before, we will side-step the equilibrium selection question and confine our attention to the pricing implications of the sanitization strategy in the multi-period setting.

Denote by θ_t the probability that the manager learns of the success or otherwise of a project by date t . The sanitization strategy in the multi-period setting consists of the manager disclosing all the projects that are known to have succeeded and suppressing all known failures. We maintain the independence assumptions across projects on whether it succeeds or not, and whether the manager learns of its outcome or not. Then define the sequence $\{q_t\}_{t=0}^T$ as follows.

$$\begin{aligned} q_0 &= r \\ q_t &= \frac{q_{t-1} - q_{t-1}\theta_t}{1 - q_{t-1}\theta_t} \end{aligned}$$

Since θ_t is increasing over time, q_t is decreasing over time. At date t , the

probability that the manager announces the success of a project is given by $r\theta_t$. From lemma 1, the residual uncertainty over the undisclosed projects is governed by the binomial density with success probability q_t . Figure

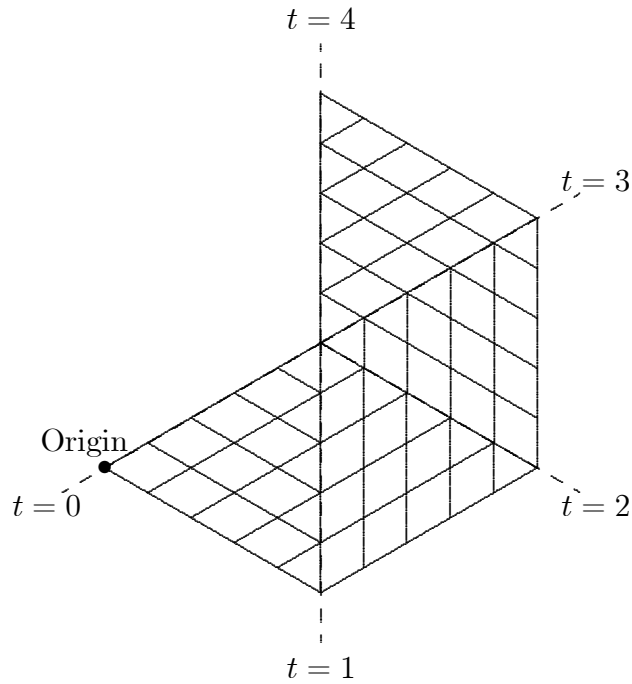


Figure 2: Pricing Spiral with Three Reporting Stages

2 depicts an example with three reporting stages (dates 1, 2 and 3) and five projects. The uncertainty structure is given by the concatenation of four binomial trees into a spiral shape - hence “pricing spiral”. As time progresses, the state of the system proceeds in the anti-clockwise direction. Between dates $t - 1$ and t , the probability of an “up” move is $r\theta_t$ while the probability of a “down” move is $1 - r\theta_t$. Each up move entails a one step jump to the centre of the spiral, while each down move means that the state of

the system maintains the same distance from the centre (measured in terms of the number of steps). At reporting stage t , the probability density over disclosed successes is binomial with success probability $r\theta_t$, and the price of the firm given s disclosed successes at date t is $u^s (q_t u + (1 - q_t) d)^{N-s}$.

The comparative statics of the information variables on the price of the firm find a natural setting in the pricing spiral. As time progresses, the probability q_t declines, implying that the market becomes more and more skeptical towards the disclosures of the manager, giving less benefit of the doubt that the manager may be uninformed. Increasingly over time, no news is taken to mean bad news, so that for the same face value disclosure, the market price is lower. Another feature of our framework highlighted by the pricing spiral is the way in which the uncertainty declines as the state of the system winds up towards the centre of the spiral. Since the system can never move away from the centre (it must either move into the centre or continue at the same distance from it), the uncertainty is greatest when it is near the edge of the spiral. We will now explore this feature and other empirical implications of our framework in more detail.

4 Points of Contact with the Empirical Literature

The most distinctive feature of our framework is the way in which information arrival affects the residual uncertainty. In order to highlight this feature, let us consider by way of contrast a world in which all information arrives *exogenously*. For instance, we may consider the case where the manager is

not strategic, and follows the policy of always disclosing fully. The lowest interim price would then be obtained when all the projects are revealed to have failed - the price being d^N . Then, all uncertainty is resolved. The same is true when all projects are revealed to have succeeded. The price will then be u^N , and there is no residual uncertainty. The greatest residual uncertainty remains when the manager is not able to reveal any successes or failures. In this case, none of the uncertainty has been resolved by the interim date. So, when information arrives exogenously, the mapping from the interim value of the firm at date 1 to the size of the residual uncertainty is a non-monotonic function. The residual uncertainty is lowest when the interim value of the firm is either at its minimum or maximum. The greatest residual uncertainty arises when the interim value of the firm is moderate.

Contrast this with the information revealed by the *strategic* manager who follows the sanitization strategy. The lowest residual uncertainty arises when the manager reveals all projects to have succeeded. However, the mapping from the interim value of the firm at date 1 to the size of the residual uncertainty is now monotonic and downward sloping. This is because the lowest interim value of the firm results when the manager reveals nothing. This is precisely the situation in which residual uncertainty is at its greatest, and suggests the following hypothesis.

Hypothesis. When information arrives through the disclosures of interested parties, the residual uncertainty is at its greatest when the news is bad and at its smallest when the news is good. When information arrives via an exogenous signal, there is no such asymmetry.

We can easily calculate the expressions for residual uncertainty that arises from our framework. To economize on notation, we conduct our discussion in terms of the three period model where the manager has a single disclosure stage at date 1. Define the *first period return* as

$$R_1 = \frac{V_1}{V_0}$$

and define the *second period return* as $R_2 = V_2/V_1$. The first period return is a random variable that takes value:

$$\frac{u^s (qu + (1 - q) d)^{N-s}}{(ru + (1 - r) d)^N}$$

with probability

$$\binom{N}{s} (r\theta)^s (1 - r\theta)^{N-s}$$

while the second period return conditional on s disclosed successes is the random variable taking value

$$\frac{u^{s+j} d^{N-s-j}}{u^s (qu + (1 - q) d)^{N-s}}$$

with probability

$$\binom{N-s}{j} q^j (1 - q)^{N-s-j}$$

From this, we have

$$\begin{aligned} \mathbb{E}(R_2^2|s) &= \frac{\mathbb{E}(u^{2j} d^{2(N-s-j)})}{(qu + (1 - q) d)^{2(N-s)}} \\ &= \left[\frac{qu^2 + (1 - q) d^2}{(qu + (1 - q) d)^2} \right]^{N-s} \end{aligned}$$

Defining

$$\chi \equiv \frac{qu}{qu + (1 - q) d}$$

we can write conditional variance as

$$\begin{aligned}\text{Var}(R_2|s) &= \left[\frac{qu^2 + (1-q)d^2}{(qu + (1-q)d)^2} \right]^{N-s} - 1 \\ &= \left(\frac{\chi u + (1-\chi)d}{qu + (1-q)d} \right)^{N-s} - 1\end{aligned}$$

Conditional variance is decreasing in s . The intuition is clear from the concatenation of pricing trees given in figure 1. As the disclosed number of successes s increases, the subsequent pricing tree is decreasing in length. The conditional variance is also increasing in the ratio u/d , since

$$\frac{\chi}{1-\chi} = \frac{q}{1-q} \left(\frac{u}{d} \right)$$

We can also express the conditional variance as a function of first period return R_1 . First period return given s disclosed successes is

$$\begin{aligned}R_1(s) &= \frac{u^s (qu + (1-q)d)^{N-s}}{(ru + (1-r)d)^N} \\ &= \left(\frac{u}{qu + (1-q)d} \right)^s \left(\frac{qu + (1-q)d}{ru + (1-r)d} \right)^N\end{aligned}$$

This gives:

$$(13) \quad s = \frac{\log R_1 + N \log \left[\frac{ru+(1-r)d}{qu+(1-q)d} \right]}{\log \left[\frac{u}{qu+(1-q)d} \right]}$$

We can write the conditional variance $\text{Var}(R_2|R_1)$ as

$$\text{Var}(R_2|R_1) = \left(\frac{\chi u + (1-\chi)d}{qu + (1-q)d} \right)^A - 1$$

where

$$(14) \quad A = \frac{N \log \left[\frac{u}{ru+(1-r)d} \right] - \log R_1}{\log \left[\frac{u}{qu+(1-q)d} \right]}$$

Thus we can state:

Theorem 4 $Var(R_2|R_1)$ is a decreasing function of R_1 .

A number of empirical regularities may be better understood by reference to this result. Black (1976)'s 'leverage hypothesis' has already been mentioned in the introduction. He documented how low stock return was associated with an increase in the subsequent return volatility, and suggested the hypothesis that the reduced proportion of equity within the total assets of the firm may be one explanation for this empirical regularity. Subsequent research has revealed that the increased leverage is too small to account for the size of the effect on volatility (Christie (1982), Schwert (1989), Figlewski and Wang (2000)), although the terminology of the 'leverage hypothesis' has now become a shorthand for the empirical regularity first discovered by Black. Although there is a large econometrics literature that attempts to model the increased return variance following low returns, the underlying microeconomics of the phenomenon remains an open issue. Theorem 4 addresses this gap.

We have also mentioned in the introduction how the leverage hypothesis may also be useful in addressing the anomalies with the Merton (1974) model for the pricing of defaultable securities. Since the payoff to a creditor is akin to having a short position in a put option on the assets of the firm, with a strike price equal to the face value of the debt, any increase in underlying volatility implies a transfer away from the creditor in favour of the equity holder. As compared to the benchmark case in which volatility is constant, the increased volatility following a bad outcome for the firm lowers the price of debt by *more* than the benchmark case, and lowers the price of equity by

less than the benchmark.

At a more speculative level, our framework may be of some help in thinking about the *value of private information*. Kosowski (2001) has examined the relative performance of U.S. mutual funds over the period 1962 - 1994 by comparing the value added of the stock picking skills of the mutual fund managers across two sub-periods - times of growth in the U.S. economy against recessionary periods. He finds that there is a much greater disparity in the mutual fund performances during recessions than during boom times. The implication is that the stock picking skills of fund managers have greater value during recessions. One way to rationalize this finding is to suppose that recessions are associated with lower interim values of the firm, and hence greater residual uncertainty over the true value of the firms. In such an environment, any private information that the fund manager can bring to the problem of stock selection will have greater value. Thus, during recessions, the differences in the value of private information across fund managers will be larger, leading to larger disparities in the performances of the fund managers. During booms, however, more information is disclosed voluntarily by the firms, implying that differences in the value of private information are lower, and performance disparities across fund managers are smaller.

One implication of Black's leverage hypothesis in an asset pricing context with risk averse investors is that investors must be compensated for the increased residual uncertainty of returns through higher expected returns. Campbell and Kyle (1993) assume this regularity as one of the building blocks in their model of asset returns. Following a low return, the subsequent return

is higher than it would have been following a high return. A consequence of this is that stock returns are negatively autocorrelated. This consequence of risk aversion for the asset pricing can be shown very simply in our framework when the investors have constant relative risk aversion - that is, when their von Neumann Morgenstern utility function is

$$(15) \quad u(c) = \frac{c^{1-\alpha}}{1-\alpha}.$$

The price of the firm at the interim date is obtained from the state prices across the realized numbers of successes at the final date. Each state price is proportional to the product of the probability of that state and the marginal utility of consumption at that state, where the constant of proportionality is chosen so that the state prices sum to 1. Since marginal utility is given by $u'(c) = c^{-\alpha}$, the state price for the outcome in which there are $s+k$ successes at the final date conditional on s disclosed successes at the interim date is

$$\frac{\binom{N-s}{k} q^k (1-q)^{N-s-k} (u^{s+k} d^{N-s-k})^{-\alpha}}{\sum_{i=0}^{N-s} \binom{N-s}{i} q^i (1-q)^{N-s-i} (u^{s+i} d^{N-s-i})^{-\alpha}}$$

Thus, the price of the firm at the interim date as a function of the disclosed number of successes s is given by:

$$\begin{aligned} & \frac{u^s \sum_{i=0}^{N-s} \binom{N-s}{i} q^i (1-q)^{N-s-i} (u^{s+i} d^{N-s-i})^{1-\alpha}}{\sum_{i=0}^{N-s} \binom{N-s}{i} q^i (1-q)^{N-s-i} (u^{s+i} d^{N-s-i})^{-\alpha}} \\ &= u^s \left[\frac{qu^{1-\alpha} + (1-q)d^{1-\alpha}}{qu^{-\alpha} + (1-q)d^{-\alpha}} \right]^{N-s} \end{aligned}$$

Defining the constant

$$(16) \quad \pi \equiv \frac{qu^{-\alpha}}{qu^{-\alpha} + (1-q)d^{-\alpha}}$$

the interim price given s can be written as

$$(17) \quad u^s (\pi u + (1 - \pi) d)^{N-s}$$

Comparing this expression with the corresponding pricing formula for the risk-neutral case (12), we see that the effect is to replace q by π . Since $\pi < q$, the effect of risk aversion is to reduce the price of the firm at the interim date. The price of the firm at the interim date with risk aversion coincides with the price in the risk-neutral case where the posterior probability of success has been reduced from q to π . From (12) the expected payoff at the final date conditional on s is $u^s (qu + (1 - q) d)^{N-s}$, so that the expected second period return conditional on s is

$$E(R_2|s) = \left(\frac{qu + (1 - q) d}{\pi u + (1 - \pi) d} \right)^{N-s}$$

We note two features.

- $E(R_2|s) > 1$. Risk averse investors expect a return that is higher than the actuarially fair rate. Note from (16) that the more risk averse the investors are, the greater must be the price discount.
- The expected second period return is decreasing in s . This reflects the greater residual uncertainty following a low number of disclosed successes. The greater the residual uncertainty, the lower must be the price relative to its expected payoff in order to compensate a risk averse investor. Returns are negatively autocorrelated.

There is another set of empirical findings that may be understood better by reference to our framework. Several studies in the empirical accounting

literature have examined the price reactions to earnings announcements, and have documented several regularities in the way that the price reactions vary across firms. Grant (1980), Atiase (1985) and Freeman (1987) show that the stock return variance at the point of release of the earnings announcement is greater along the following dimensions

- the smaller is the firm
- the less predictable is the earnings series
- the fewer stories there are in the *Wall Street Journal* prior to the earnings announcement

In terms of our framework, the reaction of the firm's price to an earnings announcement corresponds to the determination of the interim price V_1 as a function of the disclosure s . In other words, the question is how the first period return $R_1 = V_1/V_0$ depends on the underlying parameters of the problem. Let us revert to our risk neutral framework in which the prices are given by the actuarially fair value of the payoffs. The first period return R_1 is the random variable that takes value:

$$\frac{u^s (qu + (1 - q) d)^{N-s}}{(ru + (1 - r) d)^N}$$

with probability

$$\binom{N}{s} (r\theta)^s (1 - r\theta)^{N-s}$$

so that the variance of R_1 is given by

$$\begin{aligned}\text{Var}(R_1) &= \text{E}(R_1^2) - 1 \\ &= \left[\frac{r\theta u^2 + (1-r\theta)\hat{d}^2}{(r\theta u + (1-r\theta)\hat{d})^2} \right]^N - 1\end{aligned}$$

where $\hat{d} = (qu + (1-q)d)$. Defining

$$\xi \equiv \frac{r\theta}{r\theta u + (1-r\theta)\hat{d}}$$

we have

$$\text{Var}(R_1) = \left(\frac{\xi u + (1-\xi)\hat{d}}{r\theta u + (1-r\theta)\hat{d}} \right)^N - 1$$

Note the following features.

- Variance of R_1 is increasing in θ . Thus, the greater is the information asymmetry between the manager and the market, the greater is the price reaction to the disclosure. To the extent that there is a greater asymmetry of information for smaller firms, this would explain the result that the stock price reaction to earnings announcements are larger for smaller firms. Another way to phrase this is to say that when θ is high, the information content of the disclosure is higher. Holthausen and Verrecchia (1988) have examined the informativeness of signals in a multivariate normal context, and reach similar conclusions.
- Alternatively, θ may be smaller for larger firms for reasons of more frequent disclosures. The multi-period “pricing spiral” of figure 2 is one way to rationalize this phenomenon. A firm that has many

disclosure stages reveals a little bit of incremental information at a time. In contrast, the disclosure of a firm that has very few disclosure stages will have a great deal more information value.

- The number of stories in the *Wall Street Journal* prior to the earnings announcement corresponds to a greater degree of public information, and hence a less severe information asymmetry between the market and the firm. There are two ways to think about this. One is through a lower θ as above. Another is to think of N being smaller.

We conclude this section on empirical issues by considering how we may define the “whisper number” on the expected disclosure of the firm. The whisper number may be defined as the disclosure s that would leave the value of the firm at the interim date unchanged from its initial value. This can be obtained from (13) by solving for the disclosure that results in $\log R_1 = 0$. Thus, the whisper number can be written as

$$(18) \quad \frac{s}{N} = \frac{\log \left[\frac{ru+(1-r)d}{qu+(1-q)d} \right]}{\log \left[\frac{u}{qu+(1-q)d} \right]}$$

The whisper number s is decreasing in q , and hence increasing in θ . This makes intuitive sense. The greater is θ , the better informed is the manager, and the greater is the scope for manipulation of news. The market is consequently less willing to give the benefit of the doubt to the manager, and sets a high hurdle for the disclosure.

5 Unbounded Number of Projects

One drawback of our framework so far is the assumption that the number of projects N is common knowledge. The residual uncertainty is bounded by N , and so gets “squeezed from the top” as the number of disclosed successes increases. This raises the natural question as to how robust our results are to extensions of the model in which the number of potential projects is unbounded, or when there is asymmetric information about N .

The question is best posed by asking about the determinants of residual uncertainty when the number of potential projects is unbounded. Does the residual uncertainty increase or decrease as the number of disclosed successes goes up? There are two forces at work. On the one hand, when the number disclosed successes is large, the signal is informative, so that the residual uncertainty could be considered small. This is the intuition that has driven our framework so far. However, there is an effect that goes in the opposite direction. The disclosure of a large number of successes could mean that it is more likely that the number of projects is large, so that even with the larger number of disclosed successes, there is an even larger number of undisclosed projects. The residual uncertainty could then be larger when there are more disclosed successes.

We will throw some light on this question by solving for the knife-edge case where the two forces exactly cancel each other out, so that the residual uncertainty remains *invariant* to the disclosure at the interim stage. The value of identifying such a knife-edge case is that we can then identify suffi-

cient conditions that will determine whether residual uncertainty is increasing or decreasing. Thus, although the formal argument will not settle the issue of how residual uncertainty changes, it provides the necessary framework for assessing how intuitively plausible are the conditions that lead to one case or the other. We will see that the sufficient conditions that ensure decreasing residual uncertainty are very plausible ones. The ultimate arbiter will be the empirical evidence.

The following thought experiment motivates the knife-edge case where the residual uncertainty is invariant to the disclosure. Let time run continuously, and index it by the unit interval $[0, 1]$. Date 0 is the initial date, and date 1 is the terminal date. Suppose that projects arrive randomly according to a Poisson process with intensity λ , and each project succeeds with probability r . Assume that the manager learns the outcome of a project with probability θ . At date $\tau \in (0, 1)$ the manager is obliged to make a disclosure. Suppose that the manager follows the sanitization strategy and discloses all known successes up to date τ . Whatever the announcement is at date τ , the Poisson arrival process ensures that future arrivals of successful projects are independent of what has been announced already. So, the conditional beliefs over future successes should not depend on the announcement at date τ .

This thought experiment suggests that we can extend our existing model to accommodate the intuition for invariant residual uncertainty by supposing that the ex ante number of projects is a random variable with a Poisson density. Thus, let us introduce a prior stage at which nature draws N according to the Poisson density with parameter λ . The probability that

the firm has N projects is

$$e^{-\lambda} \frac{\lambda^N}{N!}$$

The market does not observe the outcome of nature's draw, and prices the firm at date 0 according to the ex ante information. Thereafter, the game proceeds as in the finite game. The ex ante probability of k *successful* projects is given by

$$e^{-\lambda} \left\{ \begin{aligned} & r^k \frac{\lambda^k}{k!} + \binom{k+1}{1} r^k (1-r) \frac{\lambda^{k+1}}{(k+1)!} + \binom{k+2}{2} r^k (1-r)^2 \frac{\lambda^{k+2}}{(k+2)!} + \dots \\ & \qquad \qquad \qquad + \binom{k+j}{j} r^k (1-r)^j \frac{\lambda^{k+j}}{(k+j)!} + \dots \end{aligned} \right\}$$

where the typical term $\binom{k+j}{j} r^k (1-r)^j \frac{\lambda^{k+j}}{(k+j)!}$ denotes the probability that there are $k+j$ projects from which there are k successes. Simplifying, we have

$$\begin{aligned} & e^{-\lambda} \frac{(r\lambda)^k}{k!} \left\{ 1 + \lambda(1-r) + \frac{\lambda^2(1-r)^2}{2!} + \frac{\lambda^3(1-r)^3}{3!} + \dots \right\} \\ & = e^{-r\lambda} \frac{(r\lambda)^k}{k!} \end{aligned}$$

So that the ex ante density over the number of successful projects is Poisson with parameter $r\lambda$.

Our focus is on the conditional density at date 1. The only information available to the market is the number of disclosed successes at date 1, where the disclosure results from the sanitization strategy of the manager. Denote by $h(k|s)$ the probability of k realized successes at date 2 conditional on s *disclosed* successes at date 1. Then, we have the following result.

Theorem 5 *If the manager follows the sanitization strategy,*

$$h(s+j|s) = e^{-r\lambda(1-\theta)} \frac{(r\lambda(1-\theta))^j}{j!}$$

In other words, the residual uncertainty is invariant to the disclosure s , and is given by the Poisson density with parameter $r\lambda(1-\theta)$. We have thus identified our knife-edge case where the residual uncertainty is neither increasing nor decreasing. To prove the theorem, note that

$$\begin{aligned}
\frac{h(k+1|s)}{h(k|s)} &= \frac{h(s, k+1)}{h(s, k)} \\
&= \frac{e^{-r\lambda} \left[(r\lambda)^{k+1} / (k+1)! \right] \binom{k+1}{s} \theta^s (1-\theta)^{k+1-s}}{e^{-r\lambda} \left[(r\lambda)^k / k! \right] \binom{k}{s} \theta^s (1-\theta)^{k-s}} \\
&= \frac{r\lambda(1-\theta)}{k+1-s} \\
&= \frac{e^{-r\lambda(1-\theta)} \left[(r\lambda(1-\theta))^{k+1-s} / (k+1-s)! \right]}{e^{-r\lambda(1-\theta)} \left[(r\lambda(1-\theta))^{k-s} / (k-s)! \right]}
\end{aligned}$$

The second line follows from the fact that the manager announces s out of k true successes when he fails to observe $k-s-1$ of the successes, but observes the rest.

This result tells us that when the ex ante density over the number of successful projects is Poisson with parameter $r\lambda$, then the conditional beliefs over the undisclosed projects is also Poisson with parameter $r\lambda(1-\theta)$. In fact, Poisson densities have a closure property analogous to our result on binomial densities. The weighted average of Poisson densities with Poisson weights is itself a Poisson density. By analogy with binomial matrices introduced in

an earlier section, define the *a-Poisson matrix* as the doubly infinite array:

$$P(a) \equiv e^{-a} \begin{bmatrix} 1 & a & \frac{a^2}{2!} & \frac{a^3}{3!} & \cdots \\ 0 & 1 & a & \frac{a^2}{2!} & \cdots \\ 0 & 0 & 1 & a & \cdots \\ 0 & 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

The s th row of $P(a)$ starts with a sequence of zeros until it reaches the main diagonal. Thereafter, the entries follow the Poisson density with parameter a . Poisson matrices are triangular, and hence their multiplication is well-defined due to the fact that each entry in the product is a finite sum. We have the following result which mirrors lemma 2.

Lemma 6 $P(a)P(b) = P(a+b)$

Proof. The (i, j) th entry of $P(a)P(b)$ is given by

$$\begin{aligned} \sum_{k=i}^j e^a \frac{a^{k-i}}{(k-i)!} e^b \frac{b^{j-k}}{(j-k)!} &= \frac{e^{a+b}}{(j-i)!} \sum_{k=i}^j \binom{j-i}{k-i} a^{k-i} b^{j-k} \\ &= e^{a+b} \frac{(a+b)^{j-i}}{(j-i)!} \end{aligned}$$

which is the (i, j) th entry of $P(a+b)$.

We can make use of this lemma in deriving the ex ante density over disclosed successes by the manager. By letting $r\lambda = a+b$ and $b = r\lambda(1-\theta)$, we conclude that $a = r\lambda\theta$. Since the top row of $P(a)$ is the ex ante density over the number of disclosed successes, we conclude that this density must be Poisson with parameter $r\lambda\theta$.

Having identified the knife-edge case where the residual uncertainty is invariant to the disclosure, we motivate a sufficient condition for residual uncertainty to be decreasing in the number of disclosed successes. We can start by returning to the initial thought experiment that motivated our adoption of the Poisson density - namely, the property of independent increments embedded in the Poisson process. If there are short term constraints on the firm's capacity to take on additional projects, or if there are industry-wide or general equilibrium effects that constrain the ability of the firm to undertake new projects, it would be reasonable to suppose that the arrival rate of new projects is endogenous, and is smaller when the firm already has a large number of on-going projects. This would argue for an arrival rate λ that is dependent on the number of projects that have already arrived, and which is falling in the number of existing projects. The branch of probability theory that deals with such processes is called "birth processes", and could form the basis of a more systematic study (see Grimmett and Stirzaker (1992, ch. 6)).

Here, we will confine ourselves to finding sufficient conditions on the ex ante density over successes that will determine whether residual uncertainty is increasing or decreasing. The firm's short term capacity constraint suggests that the property of decreasing residual uncertainty (as disclosed success increases) rests on the fall in the arrival rate of new projects when the number of projects becomes large. This is the intuition that lies behind the following result. Denote by $h_s(j)$ the density over residual uncertainty conditional on s , defined as

$$h_s(j) \equiv h(s+j|s)$$

Theorem 7 *Suppose that the ex ante density $h(\cdot)$ over the number of successful projects is such that*

$$(19) \quad \frac{k \cdot h(k)}{h(k-1)}$$

is a decreasing function of k . Then, the density h_s dominates h_{s+1} in the sense of first degree stochastic dominance.

The sufficient condition identified in the theorem states that the density $h(\cdot)$ must have a slope that is steeper than a Poisson density, since for a Poisson density (19) is constant. This condition is motivated by the discussion on the short run capacity constraint of the firm. When the number of successful projects increases, the capacity constraint of the firm starts to bite, and leads to the decreasing arrival rate of new successful projects.

First degree stochastic dominance is the criterion we adopt for increasing residual uncertainty. When h_s dominates h_{s+1} , the probability mass over the undisclosed projects is spread over outcomes that are further away from the disclosure itself as we go from $s+1$ to s . We could imagine a variant of the theorem in which decrease in (19) holds only to the right of some benchmark point k_0 . What's important is that the capacity constraint starts to bite eventually. Decreasing residual uncertainty kicks in beyond k_0 .

We conclude this section by proving the theorem. For any $k > s$, we have by Bayes rule

$$\frac{h(k|s)}{h(k-1|s)} = \frac{h(k)}{h(k-1)} \cdot \frac{h(s|k)}{h(s|k-1)}$$

But $h(s|k) = \binom{k}{s} \theta^s (1 - \theta)^{k-s}$, so that

$$\frac{h(k|s)}{h(k-1|s)} = \frac{h(k)}{h(k-1)} \frac{k(1-\theta)}{k+1-s}$$

Thus

$$(20) \quad \frac{h(k|s)}{h(k-1|s)} > \frac{h(k+1|s+1)}{h(k|s+1)} \iff \frac{kh(k)}{h(k-1)} > \frac{(k+1)h(k+1)}{h(k)}$$

The right hand side of (20) holds by hypothesis. Letting $j = k - 1 - s$, and by iterating (20) we have

$$\frac{h_s(i)}{h_s(j)} > \frac{h_{s+1}(i)}{h_{s+1}(j)}$$

for any $i > j$. Summing over $i \geq j + 1$,

$$\frac{H_s(j+1)}{h_s(j)} > \frac{H_{s+1}(j+1)}{h_{s+1}(j)}$$

where $H_s(j+1) \equiv \sum_{i \geq j+1} h(i)$. Taking reciprocals, adding 1 to both sides and taking reciprocals again, we have

$$(21) \quad \frac{H_s(j+1)}{H_s(j)} > \frac{H_{s+1}(j+1)}{H_{s+1}(j)}$$

Since $H_s(0) = H_{s+1}(0) = 1$, (21) implies that $H_s(1) > H_{s+1}(1)$. We then have an argument from induction. Suppose $H_s(j) > H_{s+1}(j)$. Then (21) gives

$$\frac{H_s(j+1)}{H_s(j)} > \frac{H_{s+1}(j+1)}{H_{s+1}(j)} > \frac{H_{s+1}(j+1)}{H_s(j)}$$

which implies that

$$H_s(j+1) > H_{s+1}(j+1)$$

Thus, $H_s > H_{s+1}$, so that h_{s+1} dominates h_s in the sense of first degree stochastic dominance. This concludes the proof.

6 Concluding Remarks

The theory in this paper has been developed in the context of *corporate disclosure*, and this has motivated our choice of using the framework of verifiable reports in setting up our game. We conclude by discussing other contexts in which the disclosing party has an interest in the reactions of the market. Not all of these cases would be best dealt with by using the verifiable reports framework. For brokers' stock recommendations, for example, the regulatory constraints on the generally accepted accounting principles (GAAP) would not apply, and it would be more reasonable to employ the cheap talk framework. Morgan and Stocken (2000) examine this issue.

There are some cases for which the choice of framework is more finely balanced. Disclosures by governments is one of these cases. Sovereign risk has been notoriously difficult to capture in a formal asset pricing setting since the notion of default is even less clear than in the case of corporate default. Opportunistic behaviour on the part of the debtor cannot be ruled out, where the *willingness* to repay is more relevant than the ability to repay.

One policy response to the turbulence in international markets has been to call for increased transparency of disclosure from government and other official sources, as well as major market participants. A series of initiatives are under way from multilateral organizations towards greater transparency (see BIS (1999), IMF (1998)). The theory presented in this paper could be seen as one way to formalize the notion that *uncertainty increases during a crisis*. To the extent that governments and monetary authorities have an

interest in the reactions of the market towards a particular set of outcomes, its disclosure policy will be necessarily influenced by this. The analogy between the disclosures by governments and official bodies on the one hand, and the accounting disclosures by firms on the other is only as strong as the assumption that disclosures by governments are verifiable. To the extent that the analogy can be pushed further, the absence of news is seen as bad news by sophisticated market participants, and it has the effect of increasing uncertainty.

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APPENDIX A

In this appendix we show two examples of equilibrium where the manager does not follow the sanitization strategy. The first example is where the manager discloses a failure in equilibrium. Figure 3 illustrates the argument.

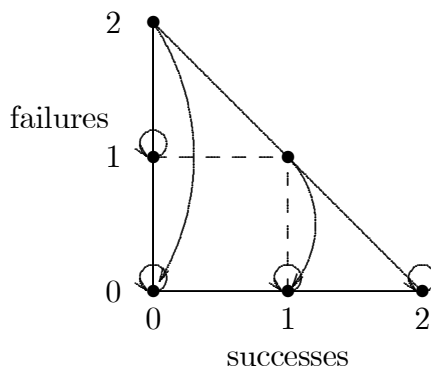


Figure 3: Equilibrium Strategy with Reported Failure

The solid dots in figure 3 represent the possible types of the manager, and the arrows represent the disclosures of the respective types. Every type except $(0, 1)$ sanitizes. Type $(0, 1)$, however, reveals his type truthfully. Denote $\rho(s, f) = u^s d^f (ru + (1 - r)d)^{N-s-f}$, the full-revelation value for type (s, f) . Consider the following pricing rule chosen by the market

failures	2	$\rho(0, 2)$		
	1	$\rho(0, 1)$	$\rho(1, 1)$	
	0	$V_1(0, 0)$	$V_1(1, 0)$	$\rho(2, 0)$
		0	1	2
		successes		

Table 6

where $V_1(0, 0)$ and $V_1(1, 0)$ are the best replies by the market against the disclosure strategy in figure 3. Thus, $V_1(0, 0)$ is the convex combination of

$\rho(0,0)$ and $\rho(0,2)$ weighted by the posterior probability of types $(0,0)$ and $(0,2)$ respectively. $V_1(1,0)$ is similarly a convex combination of $\rho(1,0)$ and $\rho(1,1)$. In particular,

$$\begin{aligned} V_1(0,0) &= \frac{(1-\theta)^2 \rho(0,0) + \theta^2 (1-r)^2 \rho(0,2)}{(1-\theta)^2 + \theta^2 (1-r)^2} \\ V_1(1,0) &= \frac{(1-\theta)r\rho(1,0) + \theta^2 r(1-r)\rho(1,1)}{(1-\theta)r + \theta^2 r(1-r)} \end{aligned}$$

Now, consider a set of parameter values such that

$$(22) \quad \rho(0,1) = V_1(0,0)$$

For example, when $d = 1, u = 2$ and $r = 0.5$, this equality holds when θ solves

$$4(1-\theta)^2(\rho(0,0) - \rho(0,1)) + \theta^2(\rho(0,2) - \rho(0,1)) = 0$$

for which there is a root at $\theta \approx 0.71$. Given (22), we claim that the reporting strategy in figure 3 and the valuation rule in table 6 constitute a sequential equilibrium. The construction relies on the fact that $(0,1)$ is indifferent between telling the truth and sanitizing, and chooses to tell the truth. The market's reaction validates the indifference.

First consider the market's valuation rule in table 6. We have already noted that $V_1(0,0)$ and $V_1(1,0)$ constitute the best reply for the market against the disclosure strategy of the manager. The disclosures $(2,0)$ and $(0,1)$ are truthful and fully revealing, so that the best replies are $\rho(2,0)$ and $\rho(0,1)$ respectively. The remaining cells in the table are for disclosures

that receive zero probability in the manager's reporting strategy. The values $\rho(0, 2)$ and $\rho(1, 1)$ are supported by the off-equilibrium belief that the types $(0, 2)$ and $(1, 1)$ have "trembled" and have disclosed truthfully by mistake. Now consider the reporting strategy of the manager in figure 3. Since $V_1(1, 0) > \rho(1, 1)$ the suppression of the one failure by type $(1, 1)$ is optimal given the valuation rule. For type $(0, 2)$, since $V_1(0, 0) = \rho(0, 1) > \rho(0, 2)$, he cannot do better than to suppress both failures. Types $(0, 0)$, $(1, 0)$ and $(2, 0)$ must report truthfully, since their feasible set of disclosures are singletons. This just leaves type $(0, 1)$. Since $\rho(0, 1) = V_1(0, 0)$, type $(0, 1)$ cannot do better than to reveal himself truthfully. Hence, the reporting strategy in figure 3 is a best reply against the valuation rule in table 6. Finally, the beliefs that underlie the values in table 6 can be obtained as the limit of a sequence as ε tends to zero of full support beliefs in which types $(0, 2)$ and $(1, 1)$ reveal themselves by mistake with probability ε . Hence, the strategies form a sequential equilibrium.

In the second example, we will exhibit a pricing rule that violates monotonicity. Let $N = 2$, so that the type space is the same as in the first example. All types except two sanitize. The two types that do not are $(1, 0)$ and $(1, 1)$. For them, the reports are

$$\begin{aligned} (1, 0) &\mapsto (0, 0) \\ (1, 1) &\mapsto (0, 0) \end{aligned}$$

The market's pricing strategy is

	2	$\rho(0, 2)$		
failures	1	$\rho(0, 1)$	$\rho(1, 1)$	
	0	$V_1(0, 0)$	$\rho(1, 1)$	$\rho(2, 0)$
		0	1	2
		successes		

Table 6

where $V_1(0, 0)$ is the best reply to the manager's reporting strategy. This example turns on the idea that no-one reports $(1, 0)$ in equilibrium. The market interprets this report as arising from a mistake by $(1, 1)$. The off-equilibrium beliefs for $(0, 1)$ and $(0, 2)$ are that they come from truthful revelation by types $(0, 1)$ and $(0, 2)$ by mistake. It turns out that for large enough r , pooling is optimal for type $(1, 0)$. Specifically, $V_1(0, 0) = V/D$ where $V = \theta^2(1-r)^2\rho(0, 2) + (1-\theta)\theta(1-r)\rho(0, 1) + (1-\theta)^2\rho(0, 0) + 2\theta^2r(1-r)\rho(1, 1) + (1-\theta)\theta r\rho(1, 0)$ and $D = \theta^2(1-r)^2 + (1-\theta)\theta(1-r) + (1-\theta)^2 + 2\theta^2r(1-r) + (1-\theta)\theta r$. For $u = 2, d = 1, \theta = 0.9$, there is a unique r (≈ 0.811) that solves

$$V_1(0, 0) = \rho(1, 1).$$

For any r greater than this, $V_1(0, 0) > \rho(1, 1) = V_1(1, 0)$. Thus, there is an open set of parameters for which the above strategies form an equilibrium, and the pricing rule violates monotonicity. Unlike the first example, this is a case where a small perturbation of the payoffs will not eliminate the equilibrium. It shows that robust departures from sanitization are possible in our model, and confirms the difficulty of tying down beliefs in a sequential game with incomplete information. The author records his debt to a referee for suggesting this counterexample to sanitization.

APPENDIX B

Proof of lemma 1. When $h(s, k)$ is positive, it is the product of two numbers - the probability that the *realized* number of successes is k , and the probability that s of these successes is observed at the interim date. Thus, for $s \leq k$,

$$\begin{aligned} h(s, k) &= \binom{N}{k} r^k (1-r)^{N-k} \cdot \binom{k}{s} \theta^s (1-\theta)^{k-s} \\ &= \frac{N!}{(N-k)! s! (k-s)!} r^k (1-r)^{N-k} \theta^s (1-\theta)^{k-s} \end{aligned}$$

Then,

$$\begin{aligned} \frac{h(s, k)}{h(s, k-1)} &= \frac{\frac{N!}{(N-k)! s! (k-s)!} \cdot r^k (1-r)^{N-k} \theta^s (1-\theta)^{k-s}}{\frac{N!}{(N-k+1)! s! (k-s-1)!} \cdot r^{k-1} (1-r)^{N-k+1} \theta^s (1-\theta)^{k-s-1}} \\ &= \frac{N-k+1}{k-s} \cdot \frac{r(1-\theta)}{1-r} \\ &= \frac{\binom{N-s}{k-s} (r(1-\theta))^{k-s} (1-r)^{N-k}}{\binom{N-s}{k-s-1} (r(1-\theta))^{k-s-1} (1-r)^{N-k+1}} \\ &= \frac{\binom{N-s}{k-s} \left[\frac{r(1-\theta)}{r(1-\theta)+(1-r)} \right]^{k-s} \left[\frac{1-r}{r(1-\theta)+(1-r)} \right]^{N-k}}{\binom{N-s}{k-s-1} \left[\frac{r(1-\theta)}{r(1-\theta)+(1-r)} \right]^{k-s-1} \left[\frac{1-r}{r(1-\theta)+(1-r)} \right]^{N-k+1}} \\ &= \frac{\binom{N-s}{k-s} q^{k-s} (1-q)^{N-k}}{\binom{N-s}{k-s-1} q^{k-s-1} (1-q)^{N-k+1}} \end{aligned}$$

where

$$q = \frac{r(1-\theta)}{(1-r) + r(1-\theta)} = \frac{r-r\theta}{1-r\theta}$$

This proves lemma 1.

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² see www.whispernumber.com, and www.earningswhispers.com

³ The second example is due to a referee.

⁴ This type of pooling has been examined by Lewis and Sappington (1993), Austen-Smith (1994), and Shin (1994).