Imperfect Common Knowledge and the Information Value of Prices

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Price level as signal of underlying fundamentals (output gap, marginal cost)

How does the quality of the signal depend on competition?

Rogoff (2003)

- more competitive economies
  - globalization, deregulation
  - “new economy” forces
- prices more responsive to fundamentals
- less temptation for policy makers to use demand management
- low equilibrium inflation

What is the relationship between

- degree of competition
- responsiveness of prices to fundamentals?

Is the relationship *monotonic*? *continuous*?

Ball and Romer (1990): “real rigidities”

Our story: imperfect common knowledge

- distributed information
- firms have their own “window on the world”
  - Townsend (1978,1983), Phelps (1983)
  - Sargent (1991), Kasa (2000), Pearlman-Sargent (2002)
  - Woodford (2003), Hellwig (2002), Adam (2002)

Firm's pricing rule

$$q_i = (1-\xi)\, E_i q + \xi E_i z$$

- z, nominal output gap
- $0 < \xi < 1$, decreases with competition

Average over i

$$q = (1-\xi)\, \bar{E}\,(q) + \xi \bar{E}\,(z)$$

Solving,

$$q = \sum_{k=1}^{\infty} \xi\,(1-\xi)^{k-1}\, \bar{E}^k z$$

With decentralized information processing:

$$\bar{E}\left(\bar{E}\,(z)\right) \neq \bar{E}\,(z)$$

Price inertia

- time $t \in \{1, 2, \cdots\}$
- output gap $\{z_t\}$ AR(1) Gaussian process
$$z_t = a + \phi z_{t-1} + \eta_t$$
- unconditional expectation $\mu = a/(1-\phi)$
- continuum of firms, private signal
$$x_{it} = z_t + \varepsilon_{it}$$
- $\varepsilon_{it} \sim N\left(0, \sigma_\varepsilon^2\right)$, i.i.d.
- firm i's information set at t
$$\{\mu, x_{i1}, x_{i2}, \cdots, x_{it}\}$$

$$E_{it}\begin{bmatrix} z_1 \\ \vdots \\ z_t \end{bmatrix} = \begin{bmatrix} \mu \\ \vdots \\ \mu \end{bmatrix} + V_{zx}V_{xx}^{-1}\begin{bmatrix} x_{i1}-\mu \\ \vdots \\ x_{it}-\mu \end{bmatrix}$$

$$E_{it}\begin{bmatrix}\mu\\ z_1\\ \vdots\\ z_t\end{bmatrix} = \left[\begin{array}{c|ccc}1 & 0 & \cdots & 0\\ \hline c_{1t} & & & \\ \vdots & & V_{zx}V_{xx}^{-1} & \\ c_{tt} & & & \end{array}\right]\begin{bmatrix}\mu\\ x_1\\ \vdots\\ x_t\end{bmatrix}$$

average over i

$$\bar{E}_t\begin{bmatrix}\mu\\ z_1\\ \vdots\\ z_t\end{bmatrix} = \left[\begin{array}{c|ccc}1 & 0 & \cdots & 0\\ \hline c_{1t} & & & \\ \vdots & & V_{zx}V_{xx}^{-1} & \\ c_{tt} & & & \end{array}\right]\begin{bmatrix}\mu\\ z_1\\ \vdots\\ z_t\end{bmatrix}$$

$$B_t \equiv \left[\begin{array}{c|ccc}1 & 0 & \cdots & 0\\ \hline c_{1t} & & & \\ \vdots & & V_{zx}V_{xx}^{-1} & \\ c_{tt} & & & \end{array}\right]$$

- B_t is transition matrix of Markov chain over $\{\mu, z_1, \cdots, z_t\}$
- μ is absorbing state, all other states transient.

$$V_{zz}V_{xx}^{-1} = E \begin{bmatrix} \frac{\lambda_1}{\lambda_1+\sigma_\varepsilon^2} & & \\ & \ddots & \\ & & \frac{\lambda_t}{\lambda_t+\sigma_\varepsilon^2} \end{bmatrix} E'$$

$$B_t^k = \begin{bmatrix} 1 & 0 \\ \left(\sum_{i=0}^{k-1} \left(V_{zx}V_{xx}^{-1}\right)^i\right) c & \left(V_{zx}V_{xx}^{-1}\right)^k \end{bmatrix}$$

$$\bar{E}_t^k \begin{bmatrix} \mu \\ z_1 \\ \vdots \\ z_t \end{bmatrix} \longrightarrow \begin{bmatrix} \mu \\ \mu \\ \vdots \\ \mu \end{bmatrix} \quad \text{as } k \longrightarrow \infty$$

As $\xi \longrightarrow 0$, aggregate price

$$q_t = \sum_{k=1}^{\infty} \xi \left(1 - \xi\right)^{k-1} \bar{E}^k z_t$$

becomes extremely sluggish.

As economy becomes more competitive,

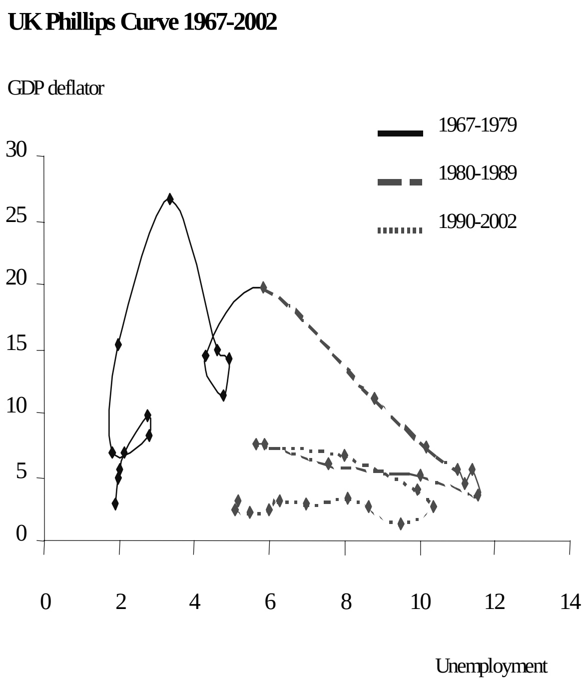
- ξ becomes smaller
- q_t is weighted average of $\bar{E}^k z_t$ with more weight on higher k
- in the limit as $\xi \to 0$, $q_t \to \mu$.
- common knowledge benchmark: $q_t = z_t$
- perfect competition benchmark: $q_t = \bar{E}\left(z_t\right) = z_t$

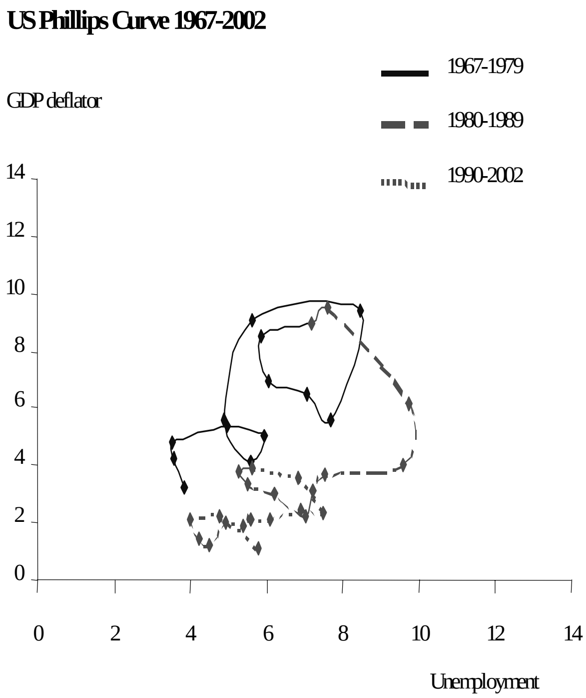
Relationship between responsiveness of prices and competition is highly discontinuous

Perfect competition $\neq$ *competitive limit of imperfect competition*

Challenges for monetary policy in 1990s

- Strong demand, surging asset prices but quiescent inflation





- Early identification of imbalances key, but...
  - hard when Phillips curve flattens
  - exacerbated by noise (e.g. beneficial supply shocks)

General Static Model

- $z_i = z + \varepsilon_i$
- state $\omega \equiv (z, \varepsilon_1, \varepsilon_2, \cdots, \varepsilon_N)$
- finite state space Ω
- Common prior ϕ over Ω

Firm i's matrix of conditional beliefs:

$$B_i \equiv \begin{bmatrix} - & b_i(1) & - \\ - & b_i(2) & - \\ & \vdots & \\ - & b_i(|\Omega|) & - \end{bmatrix}$$

$b_i(j)$: firm i's posterior conditional on state j

$$E_i f = B_i f$$

Matrix of average conditional beliefs, B:

$$\bar{E} f = B f$$

B defines Markov chain over Ω

Equilibrium price

$$
\begin{aligned}
q &= \xi \sum_{i=0}^{\infty} \left(\left(1-\xi \right) B \right)^{k} Bz \\
&= \xi \left(I - \left(1-\xi \right) B \right)^{-1} Bz \\
&= MBz
\end{aligned}
$$

Pricing rule under common knowledge:

$$q = z$$

Two differences

- $Bz \neq z$ (but $Bz \simeq z$ for small noise ε)
- M is more serious...
- M is stochastic matrix, “noise” in the sense of Blackwell

Prior is invariant distribution

$$\phi = \phi B$$

Even for small noise, imperfect common knowledge implies

$$B^k \longrightarrow B^\infty = \begin{bmatrix} - & \phi & - \\ - & \phi & - \\ & \vdots & \\ - & \phi & - \end{bmatrix}$$

$$\text{As } \xi \longrightarrow 0, \qquad q \longrightarrow B^\infty z$$

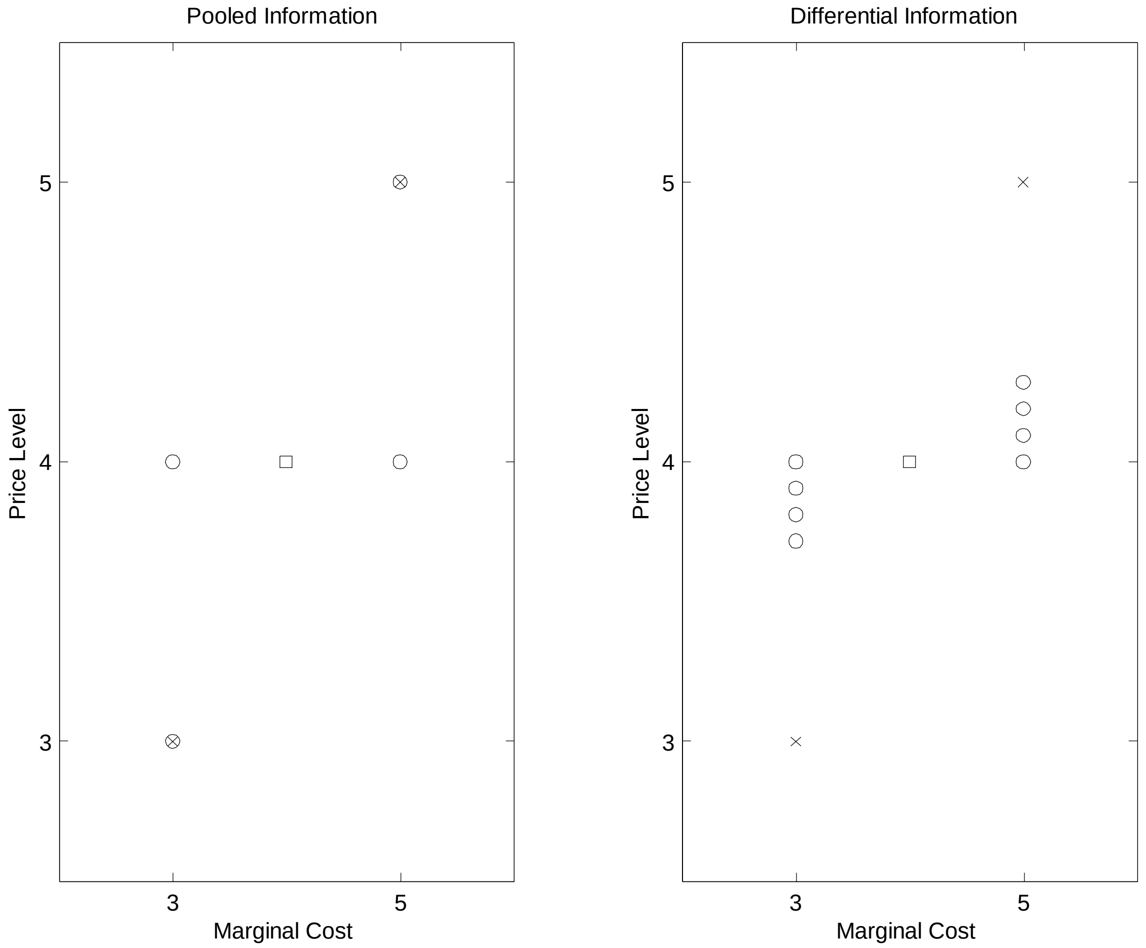
- B corresponds to reversible Markov chain
  - essentially, symmetric
  - diagonalizable
  - degradation of information depends on second largest eigenvalue

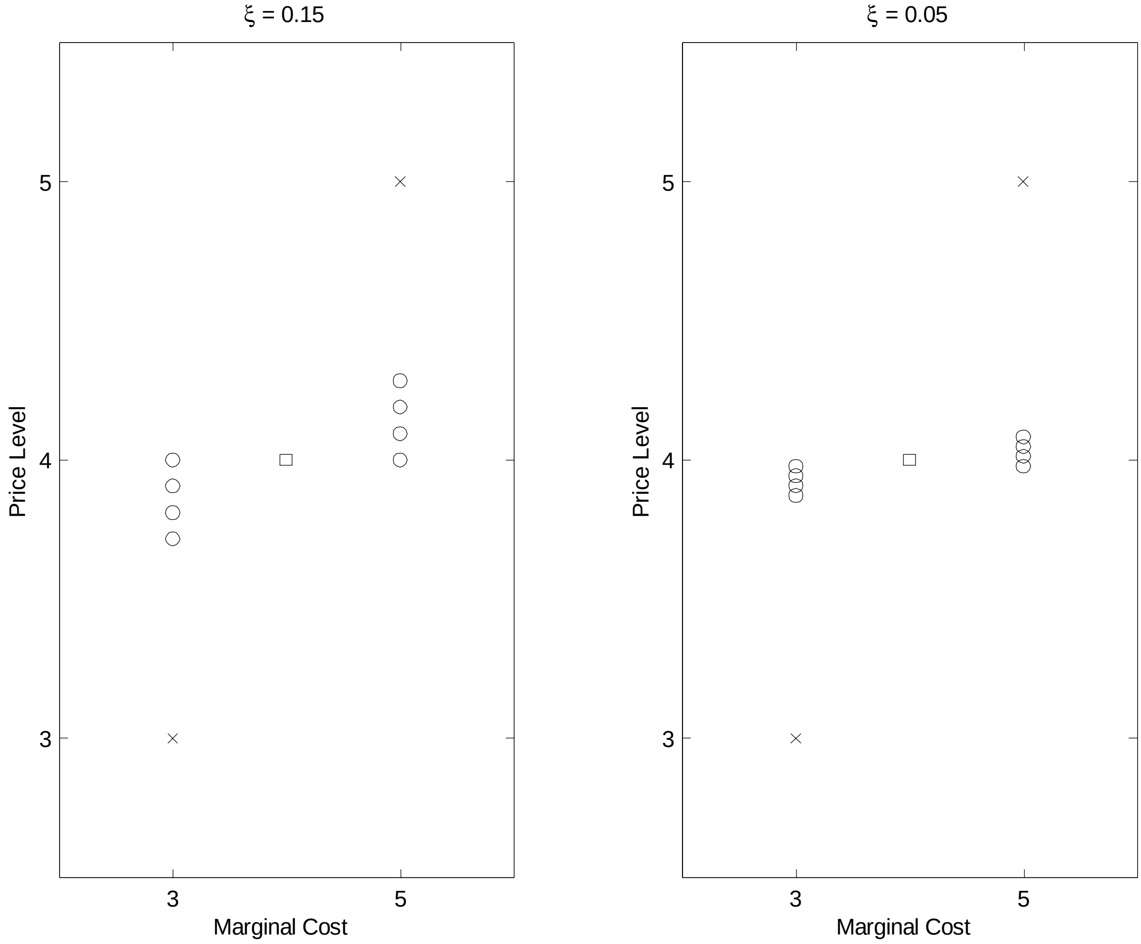
Example.

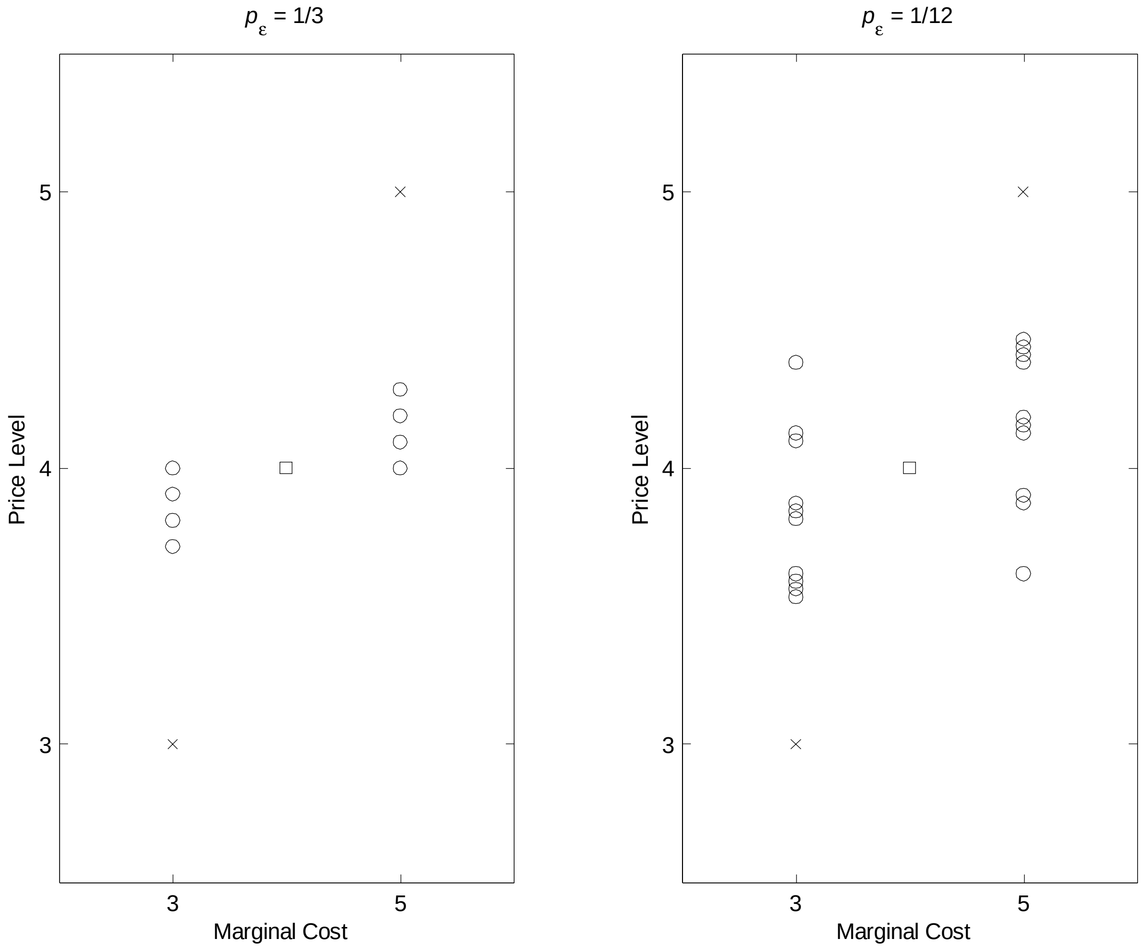
- Three firms.
- Marginal cost is 3 or 5 with equal prob.
- Firm receives signal with margin of error, 2.

$\Rightarrow$ 54 states (2 realizations for cost, 3^3 configurations of signals)

State	z	ϵ_1	ϵ_2	ϵ_3	Probability
1	z_H	ϵ	ϵ	ϵ	$p_H p_\epsilon^3$
2	z_H	ϵ	ϵ	0	$p_H p_\epsilon^2(1-2p_\epsilon)$
3	z_H	ϵ	ϵ	$-\epsilon$	$p_H p_\epsilon^3$
4	z_H	ϵ	0	ϵ	$p_H p_\epsilon^2(1-2p_\epsilon)$
5	z_H	ϵ	0	0	$p_H p_\epsilon(1-2p_\epsilon)^2$
6	z_H	ϵ	0	$-\epsilon$	$p_H p_\epsilon^2(1-2p_\epsilon)$
7	z_H	ϵ	$-\epsilon$	ϵ	$p_H p_\epsilon^3$
8	z_H	ϵ	$-\epsilon$	0	$p_H p_\epsilon^2(1-2p_\epsilon)$
9	z_H	ϵ	$-\epsilon$	$-\epsilon$	$p_H p_\epsilon^3$
10	z_H	0	ϵ	ϵ	$p_H p_\epsilon^2(1-2p_\epsilon)$
		$\dots$			
18	z_H	0	$-\epsilon$	$-\epsilon$	$p_H p_\epsilon^2(1-2p_\epsilon)$
19	z_H	$-\epsilon$	ϵ	ϵ	$p_H p_\epsilon^3$
		$\dots$			
27	z_H	$-\epsilon$	$-\epsilon$	$-\epsilon$	$p_H p_\epsilon^3$
28	z_L	ϵ	ϵ	ϵ	$p_L p_\epsilon^3$
		$\dots$			
54	z_L	$-\epsilon$	$-\epsilon$	$-\epsilon$	$p_L p_\epsilon^3$


Pooled Information
Differential Information
Price Level
Marginal Cost
3
4
5
3
5


ξ = 0.15
Price Level
5
4
3
3
5
Marginal Cost
ξ = 0.05
Price Level
5
4
3
3
5
Marginal Cost



p_ε = 1/3
p_ε = 1/12
Price Level
Marginal Cost
5
4
3
3
5