Welfare Effects of Public Information

Stephen Morris
Yale University

Hyun Song Shin
London School of Economics

Disproportionate impact of public information relative to face value.

- – market "overreaction"
  – strategic effects

$\Longrightarrow$ Any *noise* in public info. has disproportionate impact.

- – official pronouncements
  – economic statistics

Without private information, greater precision of public information always desirable.

With private information, the welfare effect of better public information is ambiguous.

$\Longrightarrow$ In a highly sensitized world, private information *crowds out* public information.

Principal and n agents

Agents' Payoffs

$$u_i = -\left(a_i - \theta\right)^2 - \rho\left(L_i - \bar{L}\right)$$

$$L_i = \frac{1}{n}\sum_{j=1}^{n}\left(a_j - a_i\right)^2$$

$$\bar{L} = \frac{1}{n}\sum_{j=1}^{n} L_i$$

Principal's Payoff

$$W = \frac{1}{n}\sum_{j} u_j = -\frac{1}{n}\sum_{j}\left(a_j - \theta\right)^2$$

Case I: Perfect Information

$$a_i = (1 - r)\,\theta + \frac{r}{n-1}\sum_{\{j|j\neq i\}} a_j$$

where

$$r = \frac{\rho\,(n-1)\,(n-2)}{n^2 + \rho\,(n-1)\,(n-2)}$$

$\Longrightarrow$

$$a_1 = a_2 = \cdots = a_n = \theta$$

No conflict of interest

Case II: Public Information only

- θ uniform prior (improper)
- *public* signal

$$y = \theta + \eta, \quad \eta \sim N\left(0, \sigma_\eta^2\right)$$

First order condition

$$a_i = (1 - r)\,\mathrm{E}\,(\theta|\, y) + \frac{r}{n-1} \sum_{\{j|j \neq i\}} \mathrm{E}\,(a_j|\, y)$$

but

$$\mathrm{E}\left(a_i \middle| y\right) = a_i\left(y\right)$$

$\Longrightarrow$

$$a_1\left(y\right) = a_2\left(y\right) = \cdots a_n\left(y\right) = y$$

Welfare

$$\begin{aligned}\mathrm{E}\left(W \middle| \theta\right) &= -\mathrm{E}\left[\left(y-\theta\right)^2 \middle| \theta\right] \\ &= -\sigma_\eta^2\end{aligned}$$

More precise public information is always better

General Case

- *private* signal

$$x_i = \theta + \varepsilon_i, \quad \varepsilon_i \sim N\left(0, \sigma_\varepsilon^2\right)$$

- Information set $\mathcal{I}_i = (y, x_i)$

First order condition

$$a_i = (1 - r)\,\mathrm{E}\left(\theta | \mathcal{I}_i\right) + \frac{r}{n-1} \sum_{\{j | j \neq i\}} \mathrm{E}\left(a_j | \mathcal{I}_i\right)$$

$$\mathrm{E}_i(\cdot) \equiv \mathrm{E}(\cdot \,|\mathcal{I}_i)$$

$$\text{name sequence } s = (s_1, s_2, \cdots, s_k)$$

$$\mathrm{E}_s^k(\cdot) \equiv \overbrace{\mathrm{E}_{s_1}(\mathrm{E}_{s_2}(\cdots \mathrm{E}_{s_k}(\cdot)\cdots))}^{k \text{ times}}$$

$$a_i = (1-r) \sum_{k=1}^{\infty} \sum_{s \in S_i^k} \left[\tfrac{r}{n-1}\right]^{k-1} \mathrm{E}_s^k(\theta)$$

$$S_i^k \equiv \left\{ (s_1, s_2, \cdots, s_k) \,\middle|\, \begin{array}{c} s_1 = i \text{ and} \\ s_m \neq s_{m+1} \text{ for all } m \end{array} \right\}$$

$$\begin{cases} \alpha = \dfrac{1}{\sigma_\eta^2} \\ \beta = \dfrac{1}{\sigma_\varepsilon^2} \end{cases}$$

$$\mathrm{E}_i\left(\theta\right) = \frac{\alpha y + \beta x_i}{\alpha + \beta}$$

$$
\begin{aligned}
\mathrm{E}_1\left(\mathrm{E}_2\left(\theta\right)\right) &= \mathrm{E}_1\left(\frac{\alpha y + \beta x_2}{\alpha + \beta}\right) \\
&= \frac{\alpha y + \beta \mathrm{E}_1\left(x_2\right)}{\alpha + \beta} \\
&= \frac{\alpha y + \beta \mathrm{E}_1\left(\theta + \varepsilon_2\right)}{\alpha + \beta} \\
&= \frac{\alpha y + \beta \mathrm{E}_1\left(\theta\right)}{\alpha + \beta} \\
&= \frac{\alpha y + \beta\left(\frac{\alpha y + \beta x_1}{\alpha + \beta}\right)}{\alpha + \beta} \\
&= \frac{\left(\left(\alpha + \beta\right)^2 - \beta^2\right) y + \beta^2 x_1}{\left(\alpha + \beta\right)^2}
\end{aligned}
$$

In general, for $s \in S_i^k$

$$E_s^k(\theta) = \left(1 - \mu^k\right) y + \mu^k x_i$$

where $\mu = \beta / (\alpha + \beta)$

$E_s^k(\theta) \to y$ as $k \to \infty$

$$\boxed{a_i = \frac{\alpha y + \beta (1 - r) x_i}{\alpha + \beta (1 - r)}}$$

or,

$$a_i = \theta + \frac{\alpha \eta + \beta (1 - r) \varepsilon_i}{\alpha + \beta (1 - r)}$$

Welfare

$$-\frac{1}{n}\sum_i \mathrm{E}\left[\left.(a_i-\theta)^2\right|\theta\right]$$

$$= -\frac{\alpha^2\mathrm{E}\left(\eta^2\right)+\beta^2\left(1-r\right)^2\left[\mathrm{E}\left(\varepsilon_i^2\right)\right]}{\left(\alpha+\beta\left(1-r\right)\right)^2}$$

$$= -\frac{\alpha+\beta\left(1-r\right)^2}{\left(\alpha+\beta\left(1-r\right)\right)^2}$$

Private information

$$\frac{\partial \mathrm{E}\left(W \,|\, \theta\right)}{\partial \beta} = \frac{\left(1-r\right)\left(\left(1+r\right)\alpha + \left(1-r\right)^2 \beta\right)}{\left(\alpha + \left(1-r\right)\beta\right)^3}$$

$$> 0$$

Public Information

$$\frac{\partial \mathrm{E}\left(W|\,\theta\right)}{\partial \alpha} = \frac{\alpha - \left(2r-1\right)\left(1-r\right)\beta}{\left(\alpha + \left(1-r\right)\beta\right)^{3}}$$

so that

$$\frac{\partial \mathrm{E}\left(W|\,\theta\right)}{\partial \alpha} \geq 0 \quad \Leftrightarrow \quad \frac{\beta}{\alpha} \leq \frac{1}{\left(2r-1\right)\left(1-r\right)}$$

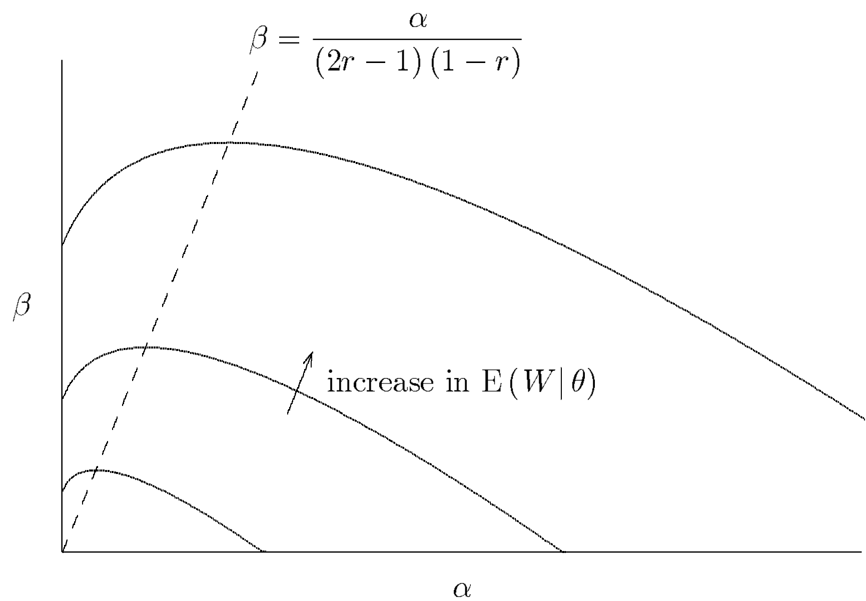

Figure 4.1: Indifference Curves for Principal

Other issues

- Linear solution method
- Alternative payoff functions for principal

Public information is a double-edged instrument

- Impact is large

- But impact of noise is large, too.